MAMT–06/MSCMT–06

June – Examination 2022

M.A./M.Sc. (Final) Examination

MATHEMATICS

(Analysis and Advanced Calculus)

Paper : MAMT-06/MSCMT-06

Time : 1½ Hours] [*Maximum Marks : **80***

Note :– The question paper is divided into two Sections A and B. Write answers as per the given instructions. Use of non-programmable scientific calculator is allowed in this paper.

Section–A **4×4=16**

(Very Short Answer Type Questions)

Note :– Answer any *four* questions. As per the nature of the questions delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries 4 marks.

1. (i) Define weak convergence of a sequence.

   (ii) Define Conjugate Space.

   (iii) State Pythagorean theorem.

   (iv) Define natural embedding.

   (v) Write Taylor's formula with Lagrange's remainder.

   (vi) Define Ortho-normal set.

   (vii) Define inner product space.

   (viii) State Polarisation identity in a Hilbert Space.

Section–B **4×16=64**

(Short Answer Type Questions)

Note :– Answer any *four* questions. Each answer should not exceed **200** words. Each question carries 16 marks.

2. State and prove Minkowski's inequality for C^n.

3. If M be a closed linear subspace of a normed linear space N and x_0 is a vector not in M, then prove that $\exists$ a function of F in conjugate space N^* s. t. F(M) = {0} and $F(x_0) \neq 0$.

4. Prove that every Hilbert space is reflexive.

5. Show that the set of unitary operators on a Hilbert space H, forms a multiplicative group.

6. Let X be a Banach space over the field K of scalars and let $f : [a, b] \to X$ and $g : [a, b] \to R$ be continuous and differentiable functions such that $||Df(t)|| \leq Dg(t)$ at each point $t \in [a, b]$. Then prove that :

$$\| f(b) - f(a) \| \leq g(b) - g(a)$$

7. State and prove global uniqueness theorem.

8. State and prove inverse function theorem.

9. Show that every compact subset of a normed linear space is bounded but its converse need not be true.