MAMT–02/MSCMT–02

June – Examination 2022

M.A./M.Sc. (Previous) Examination

MATHEMATICS

(Real Analysis and Topology)

Paper : MAMT-02/MSCMT-02

Time : 1½ Hours] [*Maximum Marks :* ***80***

Note :– The question paper is divided into two Sections A and B. Write answers as per the given instructions.

Section–A **4×4=16**

(Very Short Answer Type Questions)

Note :– Answer any *four* questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries 4 marks.

1. (i) Define measurable set.

(ii) State Weierstrass approximation theorem.

(iii) State Minkowski's inequality.

(iv) State Riesz-Fischer theorem.

(v) What do you mean by summable function ?

(vi) Define Hilbert space.

(vii) Define base for a topology.

(viii) Define embedding.

Section–B 4×16=64

(Short Answer Type Questions)

Note :– Answer any *four* questions. Each answer should not exceed **200** words. Each question carries 16 marks.

2. Let $\{E_n\}$ be a countable collection of sets of real numbers, then show that :

$$m^*\left(\bigcup_n E_n\right) \leq \sum_n m^*(E_n)$$

3. Show that every bounded measurable function f defined on a measurable set E is L-integrable.

4. Show that an orthonormal system $\{\phi_i\}$ is complete if and only if it is closed.

5. State and prove Holder's inequality.

6. Show that regularity is a topological property.

7. Prove that a subset A of Y is closed in the quotient topology τ_f relative to $f : X \to Y$ if and only if $f^{-1}(A)$ is closed in X.

8. Prove that a closed subset of a compact space is compact.

9. Prove that every filter F on a non-empty set X is contained in an ultrafilter on X.