7. Solve the integral equation and also find its resolvent kernel :

$$g(x) = f(x) + \lambda \int_{-1}^{1} \left(xt + x^2 t^2\right) g(t)\, dt$$

8. Using the method of successive approximation, solve the integral equation :

$$g(x) = 1 + \int_0^x (x - t)\, g(t)\, dt$$

taking $g_0(x) = 0$

9. Using Fredholm theory, solve :

$$g(x) = \cos 2x + \int_0^{2\pi} \sin x \cos t \;\, g(t)\, dt$$

MAMT–09/MSCMT–09

June – Examination 2022

M.A./M.Sc. (Final) Examination

MATHEMATICS

(Integral Transforms and Integral Equations)

Paper : MAMT–9/MSCMT-09

Time : 1½ Hours] [*Maximum Marks :* ***80***

Note :– The question paper is divided into two Sections A and B. Write answers as per the given instructions. Use of non-programmable Scientific Calculator is allowed in this paper.

Section–A **4×4=16**

(Very Short Answer Type Questions)

Note :– Answer any *four* questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries 4 marks.

1. (i) Define Convolution of two functions.

   (ii) State and prove first shifting property for inverse Laplace transform.

   (iii) Define Fourier cosine transform.

   (iv) Write relationship between Fourier transform and Laplace transform.

   (v) Define Fredholm integral equation.

   (vi) Define symmetric kernel.

   (vii) Define Norm of a Complex Function.

   (viii) Define Complex Hilbert Space.

Section–B **4×16=64**

(Short Answer Type Questions)

Note :– Answer any *four* questions. Each answer should not exceed **200** words. Each question carries 16 marks.

2. Find Laplace transform of the function $\sin\sqrt{t}$ and hence obtain Laplace transform of $\frac{\cos\sqrt{t}}{\sqrt{t}}$.

3. Using Laplace transform solve $ty'' + y' + 4ty = 0$ given $y(0) = 3$, $y'(0) = 0$.

4. Find the Mellin Transform of $\sin x$ and show that :

$$M^{-1}\left\{\Gamma(p)\sin\left(\frac{p\pi}{2}\right)f^*(1-p);x\right\}=\sqrt{\frac{\pi}{2}}F_s\{f(t);x\}$$

   where, $f^*(p)=M\{f(t);p\}$

5. If $H_\nu\{f(x);p\}=\int_0^\infty f(x)J_\nu(px)(xp)^{1/2}\,dx$, $p > 0$ and Re $(a) > 0$, Re $(\nu) > 0.5$ then show that :

$$H_\nu\left\{x^{\nu-\frac{1}{2}}e^{-ax};p\right\}=\frac{2^\nu\Gamma\left(\nu+\frac{1}{2}\right)p^{\nu+\frac{1}{2}}}{\sqrt{\pi}\left(a^2+p^2\right)^{\nu+\frac{1}{2}}}$$

6. Find the solution of the linear diffusion equation $\frac{\partial^2 U}{\partial x^2}=\frac{1}{k}\frac{\partial U}{\partial t}$ in a semi-infinite rod $x \geq 0$, satisfying the boundary conditions :

   (i) $U(0, t) = f(t)$, $t \geq 0$

   (ii) $U(x, t) \to 0$ as $x \to \infty$

   and the initial condition $U(x, 0) = 0$.