8. If A_{ij} is the curl of a covariant vector, prove that $A_{ij,\,k} + A_{jk,\,i} + A_{ki,\,j} = 0$.

9. Obtain the differential equations of geodesics for the metric :

$$ds^2 = f(x)dx^2 + dy^2 + dz^2 + \frac{1}{f(x)}dt^2$$

MAMT–04/MSCMT–04

June – Examination 2022

M.A./M.Sc. (Previous) Examination

MATHEMATICS

(Differential Geometry and Tensor)

Paper : MAMT–04/MSCMT-04

Time : 1½ Hours] [*Maximum Marks :* ***80***

Note :– The question paper is divided into two Sections A and B. Write answers as per the given instructions. Use of non-programmable Scientific Calculator is allowed in this paper.

Section–A **4×4=16**

(Very Short Answer Type Questions)

Note :– Answer any *four* questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries 4 marks.

1. (i) Define Osculating sphere.

   (ii) Define ruled surface.

   (iii) State Meusnier's theorem.

   (iv) Define Gaussian Curvature.

   (v) State Gauss-Bonnet theorem.

   (vi) Define Skew-symmetric tensor.

   (vii) Define Conjugate metric tensor.

   (viii) Define Einstein space.

Section–B 4×16=64

(Short Answer Type Questions)

Note :– Answer any *four* questions. Each answer should not exceed **200** words. Each question carries 16 marks.

2. Find the radii of curvature and torsion at a point of the curve $x^2 + y^2 = a^2,\ x^2 - y^2 = az$.

3. Prove that the indicatrix at a point of the surface $z = f(x, y)$ is a rectangular hyperbola if $(1 + p^2)t + (1 + q^2)r - 2pqs = 0$.

4. Show that the metric of a surface is invariant under parametric transformation.

5. Show that the curves $du^2 - (u^2 + c^2)dv^2 = 0$ from an orthogonal system on the right helicoids $\vec{r} = (u\cos v,\ u\sin v,\ cv)$.

6. Show that conjugate direction at a point P on a surface are parallel to conjugate diameters of the indicatrix at P.

7. Show that the metric of a Euclidean space, referred to cylindrical coordinates is given by $ds^2 = (dr)^2 + (rd\theta)^2 + (dz)^2$. Determine its metric tensor and conjugate metric tensor.