7. Show that :

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n$$

8. Prove that :

(i) $J_{-n}(x) = (-1)^n J_n(x)$

(ii) $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$

9. Prove the Orthogonal property :

$$\int_0^\infty e^{-x} L_m(x) L_n(x) dx = \delta_{mn} = \begin{cases} 0 & \text{if } m \neq n \\ 1 & \text{if } m = n \end{cases}$$

MAMT–03/MSCMT–03

June – Examination 2022

M.A./M.Sc. (Previous) Examination

MATHEMATICS

(Differential Equations, Calculus of Variations and Special Functions)

Paper : MAMT–03/MSCMT-03

Time : 1½ Hours] [*Maximum Marks :* ***80***

Note :– The question paper is divided into two Sections A and B. Write answers as per the given instructions.

Section–A **4×4=16**

(Very Short Answer Type Questions)

Note :– Answer any *four* questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries 4 marks.

1. (i) Classify $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$ as hyperbolic, parabolic or elliptic equation.

(ii) Write Laplace equation in polar co-ordinates.

(iii) Check whether the following boundary value problem is a Sturm-Liouville problem :

$$xy'' + y' + (x^2 + 1 + \lambda) = 0$$

$$y(0) = 0 \text{ and } y'(L) = 0$$

L is a constant such that L > 1

(iv) Define Kummer's Confluent Hypergeometric Function.

(v) Define Geodesics.

(vi) Define Bessel's function of first kind.

(vii) Write Hermite differential equation.

(viii) What is the associated Laguerre differential equation ?

Section–B 4×16=64

(Short Answer Type Questions)

Note :– Answer any *four* questions. Each answer should not exceed **200** words. Each question carries 16 marks.

2. Solve :

$$\left(\frac{d^3 y}{dx^3}\right)^2 + x\frac{d^3 y}{dx^3} - \frac{d^2 y}{dx^2} = 0$$

3. Solve $t - r\sec^4 y = 2q\tan y$ using Monge's method.

4. Find the shape of the curve on which a bead is sliding from the rest and accelerated by gravity will ship (without friction) in least time from one point to another.

5. Test for an extremal of the function :

$$F[y(x)] = \int_0^{\pi/2} \left(y'^2 - y^2\right) dx, y(0) = 0, y\left(\frac{\pi}{2}\right) = 1$$

6. Solve the Legendre's equation :

$$(1 - x^2)\frac{d^2 y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0$$