7. Show that general polynomial equation is not solvable by radicals for $n > 4$.

8. Prove that every finite dimensional vector space has an orthonormal basis.

9. Prove that the eigenvalues of a self-adjoint linear transformation are real.

MAMT–01/MSCMT–01

June – Examination 2022

M.A./M.Sc. (Previous) Examination

MATHEMATICS

(Advanced Algebra)

Paper : (MAMT-01/MSCMT-01)

Time : 1½ Hours] [*Maximum Marks : **80***

Note :– The question paper is divided into two Sections A and B. Write answers as per the given instructions. Use of non-programmable Scientific Calculator is allowed in this paper.

Section–A **4×4=16**

(Very Short Answer Type Questions)

Note :– Answer any *four* questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries 4 marks.

1. (i) If G_1 and G_2 are any two groups and H_1 and H_2 be normal subgroups respectively, then prove that $H_1 \times H_2 \nabla G_1 \times G_2$.

(ii) State diamond isomorphism theorem.

(iii) Show that every finite abelian group is solvable.

(iv) If R be a ring and M be a R-module, then prove that :

$$(-r)m = -(rm) = r(-m) \ \forall \ r \in R, m \in M$$

(v) Define Galois group.

(vi) Define column rank of a matrix.

(vii) Define Eigenvector of a linear transformation.

(viii) State Bessal's Inequality.

Section–B 4×16=64

(Short Answer Type Questions)

Note :– Answer any *four* questions. Each answer should not exceed **200** words. Each question carries 16 marks.

2. Prove that any *two* subnormal series for a group have equivalent refinements.

3. Prove that the ring of Gaussian integers is an Euclidian ring.

4. State and prove fundamental theorem on module homomorphism.

5. If V is finite dimensional vector space over a field F, then prove that V is isomorphic to vector space F^n.

6. If K be extension of field of rational numbers Q then show that any automorphism of K must leave every element of Q fixed.