MA/MSCMT-09

June - Examination 2019

M.A./M.Sc. (Final) Mathematics Examination

Integral Transforms and Integral Equations

Paper - MA/MSCMT-09

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C. Use of non-programmable scientific calculator is allowed in this paper.

Section - A $8 \times 2 = 16$

(Very Short Answer Questions)

Note: Section - A contains 8 (eight) very short answer type questions. All questions are compulsory. Each question carries 2 (two) marks and maximum word limit for each answer will be 30 words.

1) (i) Define Laplace transform. (2)

(ii) Define Fourier Cosine transform. (2)

(iii) Define Hankel transform. (2)

(iv) Define Volterra integral equation. (2)

(v) Define Singular integral equation. (2)

(vi) If $M\{f(x);p\} = F(p)$ then prove that

$$M\{f(ax);p\} = a^{-p}F(p)$$ (2)

(vii) Define norm of a complex function. (2)

(viii)Find $L^{-1}\left[\frac{pe^{-ap}}{p^2 - w^2}\right]; a > 0$ (2)

Section - B **4 × 8 = 32**

(Short Answer Questions)

Note: Section - B contains Eight Short Answer Type Questions. Examinees will have to answer any four (04) question. Each question is of 8 marks. Examinees have to delimit each answer in maximum 200 words.

2) Prove that $L\left[\int_t^{\infty} \frac{\cos u}{u}\, du\,;\, p\right] = \frac{\log(p^2+1)}{2p}$ (8)

3) Solve $(2D^2 + 3D - 2)\,y = 0,\ y(0) = 1,\ y(t) \to 0$ as $t \to \infty$ (8)

4) State and prove convolution theorem for Mellin Transform. (8)

5) Find the Hankel transform of (4 + 4)

(i) $\frac{\cos ax}{x}$

(ii) $\frac{\sin ax}{x}$

Taking $xJ_0(px)$ as kernel.

6) Solve the Fredholm integral equation of second kind (8)

$$g(x) = x + \lambda \int_0^1 (xt^2 + x^2 t)\, g(t)\, dt$$

7) Find the resolvent kernels of the following kernels. (8)

$K(x, t) = e^{x+t}, \qquad a = 0 \text{ and } b = 1$

8) Using Fredholm theory, solve (8)

$$g(x) = \cos 2x + \int_0^{2\pi} (\sin x \cos t)\, g(t)\, dt$$

9) Prove that If a kernel is symmetric, then all of its interated kernels as also symmetric. (8)

Section - C $\qquad 2 \times 16 = 32$

(Long Answer Questions)

Note: Section - C contains 4 Long answer type questions. Examinees will have to answer any two (02) questions. Each question is of 16 marks. Examinees have to answer in maximum 500 words.

10) Find the inverse Laplae transform of (4 + 4 + 4 + 4)

(i) $\dfrac{p}{(p^2 - a^2)^2}$

(ii) $\dfrac{p+1}{(p^2 + 2p + 2)^2}$

(iii) $\log\left(1 + \dfrac{1}{p^2}\right)$ or $\log\left(\dfrac{p^2+1}{p^2}\right)$

(iv) $\cot^{-1}(p+1)$

11) (i) Find the Fourier cosin transform of e^{-t^2} (8)

(ii) State and prove Parseval's Identity for Fourier transform. (8)

12) Heat is supplied at a constant rate Q per in the plane $z = 0$ to an infinite solid of conductivity K. Show that the steady temperature at a point distance r from the axis of the circular area of radius a and distance z from the plate $r = 0$ is given by $\frac{Qa}{2K}\int_0^\infty (e^{-pz} J_0(pr) J_1(ap)\, p^{-1})\, dp$

(16)

13) Find the Eigne values and Eigen functions of the homogeneous integral equation $g(x) = \lambda \int_0^1 K(x, t)\, g(t)\, dt$

where $K(x,t) = \begin{cases} x(t-1), & 0 \le x \le t \\ t(x-1), & t \le x \le 1 \end{cases}$ (16)
