MA/ MSCMT-05

June - Examination 2019

M.A. / MSc. (Previous) Mathematics Examination

Mechanics

Paper - MA/ MSCMT-05

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C. Use of non-programmable scientific calculator is allowed in this paper

Section - A $8 \times 2 = 16$

(Very Short Answer Questions)

Note: This Section - A contains 8 (eight) very short answer type questions. All questions are compulsory. Each question carries 2 (two) marks and maximum word limit for each answer will be 30 words.

1) (i) Define combined pendulum.

(ii) What do you mean by Intantaneous axis of rotation.

(iii) What do you mean by invariable line.

(iv) State principle of conservation of linear momentum.

(v) What is the degree of freedom of a single particle moving in space at any time *t*.

(vi) Define viscosity.

(vii) What do you mean by boundary surface.

(viii)Define conservative field of force.

Section - B **4 × 8 = 32**

(Short Answer Questions)

Note: This Section - B contains Eight Short Answer Type Questions. Examinees will have to answer any four (04) question. Each question is of 8 marks. Examinees have to delimit each answer in maximum 200 words.

2) Deduce the general equations of motion of a rigid body from D'Alembert's Principle when forces are finite.

3) The door of a railway carriage stands open at right angles to the length of the train when the latter starts to move with an acceleration f; the door being supposed to be smoothly hinged to the carriage and to be uniform and of breadth $2a$, show that its angular velocity, when it turned through an angle θ is $\sqrt{\left\{\frac{3f}{2a}\sin\theta\right\}}$.

4) Derive Euler's geometrical equations of motion.

5) The principal moments of inertia of a body at the centre of mass are A, 3 A, 6 A. The body is so started that its angular velocities about the axis are $3n$, $2n$, n respectively. If in the subsequent motion under no forces w_1, w_2, w_3 denote the angular velocities about the principal axis at time t, prove that $w_1 = 3w_3 = \frac{9n}{\sqrt{5}} \sec hu$ and

$w_2 = 3n \tan h\, u$ where $u = 3nt + \frac{1}{2}\log 5$.

6) Use Lagrange's equations to find the equation of motion of a simple pendulum.

7) Prove that equation of continuity due to cylindrical symmetry is given by $\frac{\partial \rho}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (\rho r^2 q_r) = 0$ Where q_r is the velocity in radial direction.

8) Show that $\frac{x^2}{a^2} f(t) + \frac{y^2}{b^2} \phi(t) + \frac{z^2}{c^2} \psi(t) = 1$ where $f(t) . \phi(t) . \psi(t) = 1$ is a possible form of boundary surface.

9) Determine the image of a source of strength m at a point with respect to the circle of radius '*a*'.

Section - C **2 × 16 = 32**

(Long Answer Questions)

Note: This Section - C contains 4 Long answer type questions. Examinees will have to answer any two (02) questions. Each question is of 16 marks. Examinees have to answer in maximum 500 words.

10) Two equal uniform rods *AB* and *AC* are freely hinged at A and rest in a straight line on a smooth table. *A* blow is struck at it perpendicular to the rods; show that the kinetic energy generated is $\frac{7}{4}$ times, what it would be if the rods were rigidly fastened together at *A*.

11) A symmetrical top is set in motion on a rough horizontal plane with an angular motion about its axis of figure, the axis being inclined at an angle *i* to the vertical. Show that between the greatest approach to and recess from the vertical, the centre of gravity describes an arc $h \tan^{-1}\left(\frac{\sin i}{p - \cos i}\right)$ where *p* and *h* have their usual meanings.

12) A particle moves in a straight line with central acceleration μ, x between two points x_0 and x_1, in the prescribed time $t_1 - t_0$ Show that Hamilton's principle function S is

$$\frac{\sqrt{\mu}\,[\,(x_1^2 + x_0^2)\cos(t_1 - t_0)\sqrt{\mu} - 2x_1\,x_0\,]}{2\sin(t_1 - t_0)\sqrt{\mu}}$$

13) A portion of homogenous fluid is confined between two concentric spheres of radii A and a, and is attracted towards their centre by a force varying inversely as the square of the distance. The inner spherical surface is suddenly annihilated and when the radii of the inner and outer surface of the fluid are r and R the fluid impinges on a solid ball concentric with these surfaces, prove that the impulsive pressure at any point of the ball for different values of R and r varies as $\left[(a^2 - r^2 - A^2 + R^2)\left(\frac{1}{r} - \frac{1}{R}\right)\right]^{1/2}$
