MA/MSCMT-04

June - Examination 2019

M.A. / M.Sc. (Previous) Mathematics

Examination

Differential Geometry and Tensors

Paper - MA/MSCMT-04

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C. Use of non-programmable scientific calculator is allowed in this paper.

Section - A **$8 \times 2 = 16$**

(Very Short Answer Questions)

Note: This Section - A contains 8 (eight) very short answer type questions. All questions are compulsory. Each question carries 2 (two) marks and maximum word limit for each answer will be 30 words.

1) (i) Write the equation of Tangent plane to a Ruled surface.

(ii) Write the equation of Osculating plane in Cartesian coordinates.

(iii) State Existence and Uniqueness theorem.

(iv) Define Geodesics.

(v) Write the statement of Meunier's theorem.

(vi) Write a formula to find principal radii through a point of the surface $z = f(x, y)$.

(vii) State Gauss-Bonnet theorem.

(viii) Define Quotient law of Tensors.

Section - B $4 \times 8 = 32$

(Short Answer Questions)

Note: This Section - B contains Eight Short Answer Type Questions. Examinees will have to answer any four (04) question. Each question is of 8 marks. Examinees have to delimit each answer in maximum 200 words.

2) Find the lines that have four point contact at (0, 0, 1) with the surface $x^4 + 3xyz + x^2 - y^2 - z^2 + 2yz - 3xy - 2y + 2z = 1$

3) Find the osculating plane at the point t on the helix $x = a\cos t,\ \ y = a\sin t,\ \ z = ct$.

4) Find the radii of curvature and torsion at a point of the curve $x^2 + y^2 = a^2,\ x^2 - y^2 = az$

5) Prove that the generators of a developable surface are tangents to curve.

6) Prove that on the surface $z = f(x, y)$ (Monge's form) the equations of asymptotic lines are $rdx^2 + 2s\, dx\, dy + t\, dy^2 = 0$.

7) Prove that the law of transformation of a contravariant vector is transitive.

8) Show that :

(i) $(g_{hj}\,g_{ik} - g_{hk}\,g_{ij})\,g^{hj} = (\mathrm{N} - 1)g_{ik}$,

(ii) $g_{ij}\,g_{kl}\,dg^{ik} = -\,dg_{jl}$

9) If A_{ij} is the curl of a covariant vector; prove that

$A_{ij,k} + A_{jk,i} + A_{ki,j} = 0$

Section - C **2 × 16 = 32**

(Long Answer Questions)

Note: This Section - C contains 4 Long answer type questions. Examinees will have to answer any two (02) questions. Each question is of 16 marks. Examinees have to answer in maximum 500 words.

10) Find the inflexional tangent at (x_1, y_1, z_1) on the surface $y^2z = 4ax$.

11) Prove that the metric of a surface is invariant under parametric transformation.

12) State and prove Meunier's theorem.

13) State and prove the Schur's theorem.