MA/ MSCMT-03

June - Examination 2019

M.A. / M.Sc. (Previous) Mathematics Examination

Differential Equations, Calculus of Variations and Special Functions

Paper - MA/ MSCMT-03

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C. Use of non-programmable scientific calculator is allowed in this paper.

Section - A **$8 \times 2 = 16$**

(Very Short Answer Questions)

Note: This Section - A contains 8 (eight) very short answer type questions. All questions are compulsory. Each question carries 2 (two) marks and maximum word limit for each answer will be 30 words.

1) (i) Write down the Riccati's Equation.

(ii) Write Monge's subsidiary equation for $x^2 r + 2xys + y^2 t = 0$

(iii) Define Linear Functionals.

(iv) Write Bessel's Differential equation.

(v) Find the dimension of the following differential equation.

$$2x^3 \frac{d^2y}{dx^2} = \left(y - x\frac{dy}{dx} \right)^2$$

(vi) Find the condition for the second order partial differential equation $Rs + Ss + Tt + F(x, y, z, p, q) = 0$ is elliptic.

(vii) Write down Laplace Equation.

(viii)Check whether the boundary value problem

$y'' + \lambda y = 0;\ y'(-\pi) = 0;\ y'(\pi) = 0$ is a strum Liouville

problem for $\lambda < 0$

Section - B $\quad$ **4 × 8 = 32**

(Short Answer Questions)

Note: Section - B contains Eight Short Answer Type Questions Examinees will have to answer any four (04) question. Each question is of 8 marks. Examinees have to delimit each answer in maximum 200 words.

2) Solve $y_1 = \cos x - y\sin x + y^2$

3) Find the eigen values and Eigen functions for following boundary value problem. $y'' - 2y' + \lambda y = 0 \quad y(0) = 0, y(\pi) = 0$

4) Define Gauss's Hypergeometric series and discuss its convergence.

5) Use the method of separation of variables to solve following PDE.

$$\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$$

6) Solve in series $(1 - x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

7) Prove that if $a+b+c>0$, then

$$\lim_{x \to 0} \{(1+x)^{a+b-c}\, {}_2F_1(a, b; c; x)\} = \frac{\Gamma(c)\,\Gamma(a+b-c)}{\Gamma(a)\,\Gamma(b)}$$

8) Prove that $L_n(x) = \dfrac{e^x}{n!} \dfrac{d^n}{dx^n}(x^n e^{-x})$

9) Prove that $L_n^{\alpha}(x, y) = \displaystyle\sum_{r=0}^{n} \frac{(1+\alpha)_n (1-y)^{n-r} y^r L_r^{\alpha}(x)}{(n-r)!\,(1+\alpha)_r}$

Section - C **2 × 16 = 32**

(Long Answer Questions)

Note: This Section - C contains 4 Long answer type questions. Examinees will have to answer any two (02) questions. Each question is of 16 marks. Examinees have to answer in maximum 500 words.

10) Solve $r = a^2 t$ by Monge's method.

11) Reduce the equation $xyr - (x^2 - y^2)\,s - xyt + py - qx = 2(x^2 - y^2)$ to canonical form and hence solve it.

12) Obtain the surface of minimum area, stretched over a given closed curve C, enclosing the domain D in the xy plane.

13) Solve the Gauss Hypergeometric equation.

$$x(1-x)\frac{d^2y}{dx^2} + \{\gamma - (1+\alpha+\beta)x\}\frac{dy}{dx} - \alpha\beta y = 0$$

In series in the neighbourhood of the regular singular point

(i) $x = 0$, (ii) $x = 1$, (iii) $x = \infty$