MA/MSCMT-02

June - Examination 2019

M.A. / M.Sc. (Previous) Mathematics Examination

Real Analysis and Topology

Paper - MA/MSCMT-02

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C. Use of non-programmable scientific calculator is allowed in this paper.

Section - A **$8 \times 2 = 16$**

(Very Short Answer Questions)

Note: This Section - A contains 8 (eight) very short answer type questions. All questions are compulsory. Each question carries 2 (two) marks and maximum word limit for each answer will be 30 words.

1) (i) Define Borel set.

(ii) Define measurable function.

(iii) State Holder's inequality.

(iv) State Riesz-Fisher theorem.

(v) Write the necessary and sufficient conditions for a bounded function f defined on the interval $[a, b]$, to be L-integrable.

(vi) State Parseval's identity.

(vii) Define Topological space.

(viii)Define embedding.

Section - B **4 × 8 = 32**

(Short Answer Questions)

Note: This Section - B contains Eight Short Answer Type Questions. Examinees will have to answer any four (04) question. Each question is of 8 marks. Examinees have to delimit each answer in maximum 200 words.

2) Prove that the union of two measurable sets is a measurable set.

3) Let $< f_n >$ is a sequence of measurable functions defined on a measurable set E, then prove that
$\sup_n < f_n >$ and $\inf_n < f_n >$ are also measurable set E.

4) Show that every bounded measurable function f defined on a measurable set E is L-integrable.

5) State and prove Minkowski's inequality.

6) Prove that for a subset A of a topological space (X, τ), $\overline{A} = A \cup A'$.

7) Show that the property of a space being a Haudorff space is a hereditary property.

8) Prove that closure of a connected set is connected.

9) Prove that every filter F on a non-empty set X is contained in an ultrafilter on X.

Section - C $2 \times 16 = 32$

(Long Answer Questions)

Note: This Section - C contains 4 Long answer type questions. Examinees will have to answer any two (02) questions. Each question is of 16 marks. Examinees have to answer in maximum 500 words.

10) State and prove Weierstrass approximation theorem.

11) Prove that L_2 is a complete space.

12) (i) If E is a countable set, then show that $m^*(E) = 0$.

    (ii) Prove that a subset A of Y is closed in the quotient topology τ_f relative to $f : X \to Y$ iff $f^{-1}(A)$ is closed in X.

13) Prove that a series $\sum_{i=1}^{\infty} f_i$ of pair wise orthogonal elements in L_2 is converges iff the series of real numbers $\sum_{i=1}^{\infty} \|f_i\|^2$ is convergent.