MA/MSCMT-01

June - Examination 2019

M.A./M.Sc. (Previous) Mathematics

Examination

Advanced Algebra

Paper - MA/MSCMT-01

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C. Write answers as per the given instructions. Use of non-programmable scientific calculator is allowed in this paper.

Section - A **$8 \times 2 = 16$**

(Very Short Answer Type Questions)

Note: Answer all Questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to 30 words. Each question carries 2 marks.

1) (i) Define direct product of groups.

(ii) Define conjugate class.

(iii) Define solvable group.

(iv) Define Euclidean ring.

(v) Define algebraic field extention.

(vi) Define Galois group.

(vii) Define nullity of a linear transformation.

(viii)Define orthogonal linear transformation.

Section - B **4 × 8 = 32**

(Short Answer Type Questions)

Note: Answer any four questions. Each answer should not exceed 200 words. Each question carries 8 marks.

2) Prove that every group is isomorphic to a group of permutations.

3) Let G be a finite group and D be a set of distinct representatives g_1, g_2, ..., g_n one from each of the conjugate classes of G. Then show that

$$|G| = \sum_{g \in D} [G : N(g)] = \sum_{i=1}^{n} [G : N(gi)]$$

4) If N and G/N are solvable groups then show that G is a solvable group.

5) Let R be a Euclidean ring and a non zero, non unit a $\in$R. suppose that

$a = s_1, s_2 \ldots. s_m = s_1' \, s_2' \ldots. s_n'$, where s_i and s_j' are prime elements of R. Then show that m = n and each $s_i, 1 \leq i \leq m$ is an associate of some $s_j', 1 \leq j \leq n$ and conversely.

6) For every prime p and natural number n $\geq$ 1 , show that there exists a finite field with p^n elements.

7) Let A = [aij] be an n×n matrix over a field F, then show that

$$\det(A) = \sum_{\sigma \in s_n} \in (\sigma)\, a_{\sigma(1)1}\, a_{\sigma(2)2} \ldots.\, a_{\sigma(n)n}$$

8) Let V be an inner product space and $u, v \in V$ are arbitrary vectors of V, then prove that $|<u, v>| \leq \|u\| \, \|v\|$

9) If $t_1 : V \to V$ and $t_2 : V \to V$ are linear transformations of a finite dimensional inner product space V to itself, then prove that

$$(t_1 t_2)^* = t_2^* t_1^*$$

where t_1^* denote adjoint of t_1.

Section - C **$2 \times 16 = 32$**

(Long Answer Type Questions)

Note: Answer any two questions. You have to delimit your each answer maximum upto 500 words. Each question carries 16 marks.

10) Show that any two composition series of a group G are equivalent.

11) Let F be a field and let $p(x)$ be an arbitrary polynomial of positive degree over F. Then show that any two splitting fields of $p(x)$ are isomorphic. Also the isomorphic mapping can be so chosen that each element of F is mapped onto itself and the set of roots of $p(x)$ in one splitting field is mapped one-one onto the set of roots of $p(x)$ in another splitting field.

12) Let F be a field of characteristic zero containing all nth roots of unity. If $f(x) \in F[x]$ is solvable by radicals over F, then show that the Galois group of $f(x)$ over F is solvable.

13) Show that any two finite dimensional vector spaces over the same field are isomorphic if and only if they are of the same dimension.
