MA/MSCMT-10

June - Examination 2018

M.A./M.Sc. (Final) Mathematics Examination

Mathematical Programming

Paper - MA/MSCMT-10

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C. Write answers as per the given instructions.

Section - A **$8 \times 2 = 16$**

(Very Short Answer Type Questions)

Note: Answer **all** questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to 30 words. Each question carries 2 marks.

1) (i) Define convex function.

(ii) Define constrained optimization problem and unconstrained optimization problem.

(iii) Let m and n denotes the number of equation and decision variables respectively, then what happens when $m = n$ in a Linear programming problem (LPP)?

(iv) What is the difference between linear and nonlinear programming problems?

(v) Consider the following problem :

Minimize $z = f(X)$,

Subject to $g_j(X) \leq 0; j = 1, 2, 3, \ldots, m.$

Then write the suitable Kuhn-Tucker conditions.

(vi) Define general quadratic programming problems.

(vii) Define bounded variable problem.

(viii) Write quadratic form

$Q(x) = x_1^2 + 2x_2^2 - 7x_3^2 - 4x_1x_2 + 6x_1x_3 - 5x_2x_3$

in matrix form.

Section - B $4 \times 8 = 32$

(Short Answer Questions)

Note: Answer **any four** questions. Each answer should not exceed 200 words. Each question carries 8 marks.

2) Prove that $f(x) = 1/x$ is strictly convex for $x > 0$ and strictly concave for $x < 0$.

3) Use Lagrangian function to find the optimal solution for the following non linear problem:

Maximize $f(X) = -3x_1^2 - 4x_2^2 - 5x_3^2$

Subject to $x_1 + x_2 + x_3 = 10;$

$x_1, x_2, x_3 \geq 0$

4) Use Kuhn-Tucker conditions to determine solution for the following problem :

Min. $f(x_1, x_2) = x_1^2 + 2x_2^2 - x_1x_2$

Subject to $x_1 + x_2 \geq 8;$

$x_1, x_2 \geq 0$

5) Derive the dual of the quadratic progamming problem :

Min. $f(X) = C^T X + (1/2) X^T GX$

Subject to $AX \geq b$

Where A is an $m \times n$ real matrix and G in an $n \times n$ real positive semi definite asymmetric matrix.

6) Write the basic steps involved in duality in quadratic programming.

7) Prove that every local maximum of the general convex programming problem is its global maximum.

8) Use dynamic programming to solve the following LPP :

Min. $(x_1^2 + x_2^2 + ... + x_n^2)$

Subject to $x_1 x_2 .....x_n = b$

$x_1, x_2, ... , x_n \geq 0$

9) Solve the Dynamic programming:

Max. $Z = x_1 + 9x_2$

Subject to $2x_1 + x_2 \leq 25,$

$x_2 \leq 11,$

$x_1, x_2 \geq 0$

Section - C $\qquad$ 2 × 16 = 32

(Long Answer Questions)

Note: Answer **any two** questions. You have to delimit your each answer maximum up to 500 words. Each question carries 16 marks.

10) Solve the following integer programming problem by Branch and Bound technique

Max. $Z = x_1 + x_2$

Subject to $3x_1 + 2x_2 \leq 12,$

$x_2 \leq 2,$

$x_1, x_2 \geq 0$

11) Solve the following quadratic programming problem using Wolfe's method :

Minimize $f(x_1, x_2) = x_1^2 + 2x_2^2 - 8x_1 - 10x_2$

Subject to $x_1 + x_2 \leq 25,$

$x_1 + 2x_2 \leq 8,$

$x_1, x_2 \geq 0$

12) Solve the following quadratic programming problem by Beale's method :

Max. $f(x_1, x_2) = x_1 + x_2 - x_1^2 + x_1 x_2 - 2x_2^2$

Subject to $2x_1 + x_2 \leq 1;$

$x_1, x_2 \geq 0$

13) Find optimal solution of the convex separate programming problem.

Max. $f(x_1, x_2) = 3x_1 + 2x_2$

Such that $4x_1^2 + x_1^2 \leq 16;$

$x_1, x_2 \geq 0$