MA/ MSCMT-03

June - Examination 2018

M.A./M.Sc. (Previous) Mathematics Examination

Differential Equations, Calculus of Variations and Special Functions.

Paper - MA/ MSCMT-03

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C. Write answers as per the given instructions.

Section - A **8 × 2 = 16**

(Very Short Answer Type Questions)

Note: Answer **all** questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to 30 words. Each question carries 2 marks.

1) (i) Solve $\frac{d^2y}{dx^2} = \sec^2 y \tan y$, given $y = 0, \frac{dy}{dx} = 1$ when $x = 0$.

(ii) Write Monge's subsidiary equations for $yr + s(x - y) - tx + q - p = 0$.

(iii) Write three dimensional wave equation in cylindrical coordinates.

(iv) Write Euler's equation for $f(x, y, y')$, when it is independent of x.

(v) What is difference between variation and differentiation?

(vi) Define Kummer function.

(vii) Write generating formula for Bessel's function $J_n(x)$.

(viii) Write orthogonal properties of Laguerre's polynomials.

Section - B **4 × 8 = 32**

(Short Answer Questions)

Note: Answer **any four** questions. Each answer should not exceed 200 words. Each question carries 8 marks.

2) Solve $xz^3 dx - zdy + 2ydz = 0$.

3) Classify the equation

$$\frac{\partial^2 u}{\partial x^2} + 3\frac{\partial^2 u}{\partial y^2} + 84\frac{\partial^2 u}{\partial z^2} + 28\frac{\partial^2 u}{\partial y \partial z} + 16\frac{\partial^2 u}{\partial z \partial x} + 2\frac{\partial^2 u}{\partial x \partial y} = 0$$

4) Find all Eigen values and Eigen functions of Sturm-Liouville problem

$$y'' + \lambda y = 0; y(0) = 0, y'(\pi/2) = 0$$

5) Determine the extremal of the functional $I = \int_{-a}^{a}\left[\frac{1}{2}\mu y''^2 + \rho y\right]dx$ that satisfies the boundary conditions

$$y(-a) = 0, y'(-a) = 0, y(a) = 0, y'(a) = 0.$$

6) Derive integral representation of hypergemetric function.

7) Prove that

$$P_n\left(-\frac{1}{2}\right) = P_0\left(-\frac{1}{2}\right)P_{2n}\left(\frac{1}{2}\right) + P_1\left(-\frac{1}{2}\right)P_{2n-1}\left(\frac{1}{2}\right) + \ldots\ldots + P_{2n}\left(-\frac{1}{2}\right)P_0\left(\frac{1}{2}\right)$$

8) Prove that $H_{n+1}(x) = 2xH_n(x) - 2nH_{n-1}(x), n \geq 1$.

9) Prove that $\int_0^\infty e^{-st} L_n(t)\,dt = \frac{1}{s}\left(1 - \frac{1}{s}\right)^n$.

Section - C **2 × 16 = 32**

(Long Answer Questions)

Note: Answer **any two** questions. You have to delimit your each answer maximum up to 500 words. Each question carries 16 marks.

10) Solve by Monge's method $r - t\cos^2 x + p\tan x = 0$.

11) Solve the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subject to

the following boundary conditions $u(x,0) = u(x,b) = 0$ for

$0 \leq x \leq a; u(0,y) = 0, u(a,y) = f(y)$ for $0 \leq y \leq b$

12) Solve in Series $(x - x^2)\frac{d^2 y}{dx^2} + (1 - 5x)\frac{dy}{dx} - 4y = 0$.

13) Establish Linear relation between solutions of hyper geometric equations.
