MA/MSCMT-09

June - Examination 2018

M.A./M.Sc. (Final) Mathematics Examination

Integral Transforms and Integral Equations

Paper - MA/MSCMT-09

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C. Write answers as per the given instructions.

Section - A **$8 \times 2 = 16$**

(Very Short Answer Type Questions)

Note: Section 'A' contain Eight (08) Very Short Answer Type Questions. Examinees have a attempt all questions. Each question is of 02 marks and maximum word limit may be thirty words.

1) (i) Write Dirichlet's conditions.

(ii) State Convolution product of two functions.

(iii) Write Kernel of the Mellin Transform.

(iv) Define Hankel transform.

(v) What is Singular Integral equation?

(vi) What is Neumann series?

(vii) Define Norm of a function.

(viii) State Hilbert Schmidt theorem.

Section - B **4 × 8 = 32**

(Short Answer Type Questions)

Note: Section 'B' contains Eight Short Answer Type Questions. Examinees will have to answer any four (04) questions. Each question is of 08 marks and maximum limit is 200 words.

2) Find Inverse Laplace transform of $L^{-1}\left\{\frac{9p^2-1}{(9p^2+1)^2};t\right\}$, if

$L^{-1}\left\{\frac{p^2-1}{(p^2+1)^2};t\right\} = t\cos t.$

3) Use complex inversion formula find the inverse Laplace transform of $\frac{p}{(p+1)(p-1)^2}$

4) Solve $(D^2+9)y = \cos 2t$, if $y(0) = 1,\ y\left(\frac{\pi}{2}\right) = -1$.

5) Find Fourier transform of $f(x) = \begin{cases} 1, & |t| \le 1 \\ 0, & |t| > 1 \end{cases}$

6) Solve for $f(x)$, $\int_0^\infty f(x)\sin px\,dx = \begin{cases} 1, & 0 \le p \le 1 \\ 2, & 1 \le p \le 2 \\ 0, & p > 2 \end{cases}$

7) Show that the function $g(x) = (1+x^2)^{-3/2}$ is a solution of the Volterra integral equation. $g(x) = \frac{1}{1+x^2} - \int_0^x \frac{t}{1+x^2}\,g(t)\,dt.$

8) Convert the following differential equation into integral equation

$$\frac{d^2y}{dx^2} + \lambda xy = f(x)\,;\, y(0) = 1; y'(0) = 0$$

9) Solve the following integral equation

$$g(x) = \sin x + \lambda \int_0^{\pi/2} \sin x \cos t\; g(t)\, dt$$

Section - C **2 × 16 = 32**

(Long Answer Type Questions)

Note: Section 'C' contain 4 Long Answer Type Questions. Examinees will have to answer any two (02) questions. Each question is of 16 marks and maximum words limit is 500 words.

10) Prove that

$$M\left\{\left(1 + x^a\right)^{-b}; p\right\} = \frac{\Gamma\left(p/a\right)\Gamma\left(b - p/a\right)}{a\Gamma\left(b\right)}\,; 0 < \mathrm{Re}(p) < \mathrm{Re}(ab).$$

11) State and prove Parseval's Theorem for Hankel transform.

12) Solve by method of successive approximations:

$$g(x) = \left(\frac{3}{2}e^x - \frac{1}{2}xe^x - \frac{1}{2}\right) + \frac{1}{2}\int_0^1 tg(t)\,dt$$

13) Using Fredholm determinants, find the resolvent kernel, when $K(x, t) = xe^t, a = 0, b = 1$