

MA/MSCMT-05

June - Examination 2018

M.A./MSc.(Previous) Mathematics Examination**Mechanics****Paper - MA/MSCMT-05****Time : 3 Hours]****[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C. Write answers as per the given instructions. Use of non-programmable scientific calculator is allowed in this paper.

Section - A **$8 \times 2 = 16$**

(Very Short Answer Type Questions)

Note: Section 'A' contains Eight (08) Very Short Answer Type Questions. Examinees have to attempt all questions. Each question is of 02 marks and maximum word limit may be thirty words.

- 1) (i) Write expressions for moment of inertia (M.I.) of a circular ring of mass M and radius a about its diameter.
- (ii) Define invariable line.
- (iii) State principle of conservation of angular momentum.
- (iv) Define degree of freedom.
- (v) State principle of conservation of linear momentum.

- (vi) Explain steady flow.
- (vii) Write equation of continuity in cartesian form.
- (viii) Define boundary surface.

Section - B

$4 \times 8 = 32$

(Short Answer Type Questions)

Note: Section 'B' contains Eight Short Answer Type Questions. Examinees will have to answer any four (04) questions. Each question is of 08 marks. Examinees have to delimit each answer in maximum 200 words.

- 2) An uniaxial body is supported at its centre of mass and is rotating initially with angular velocity w about an axis perpendicular to the axis of symmetry. Prove that if a couple of constant moment l is applied above the axis of symmetry, the instantaneous axis will describe a cone whose equation referred to the axis fixed in the body, of which that of z coincides with the axis of symmetry, is
- $$2Al(x^2 + y^2) \tan^{-1} \frac{y}{x} = C(C - A) w^2 z^2.$$
- 3) Two equal rods AB and BC , each of length l , smoothly jointed at B , are suspended from A and oscillates in a vertical plane through A . Show that the periods of normal oscillations are $\frac{2\pi}{n}$ where
- $$n^2 = \left(3 \pm \frac{6}{\sqrt{7}}\right) \frac{g}{l}$$

- 4) A uniform rod, of mass $3m$ and length $2l$ has its middle point fixed and a mass m attached at one extremity. The rod when in a horizontal position is set rotating about a vertical axis through its centre with an angular velocity equal to $\sqrt{\frac{2ng}{l}}$. Show that the heavy end of the rod will fall till the inclination of the rod to the vertical is $\cos^{-1}\{\sqrt{n^2 + 1} - n\}$.
- 5) A uniform rod OA , of length $2a$, free to turn about its end O , revolves with uniform angular velocity w about its vertical OZ through O , and is inclined at a constant angle α to OZ , show that the value of α is either zero or $\cos^{-1}\left(\frac{3g}{aw^2}\right)$.
- 6) Show that for a body of revolution the maximum value of the angle between the axis of the impulsive couple acting on it and the instantaneous axis of initial motion set up by the couple in the body is $\sin^{-1}\left(\frac{C - A}{C + A}\right)$.
- 7) A circular disc, of radius a , has a thin rod pushed through its centre perpendicular to its plane, the length of the rod being equal to the radius of the disc. Show that the system can not spin with the rod vertical unless the angular velocity is greater than $\sqrt{\frac{2og}{a}}$
- 8) What arrangement of sources and sinks will give rise to the function $w = \log\left(z - \frac{a^2}{z}\right)$?

- 9) A mass of fluid is in motion so that the lines of motion lie on the surface of coaxial cylinders, show that the equation of continuity is
- $$\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial(\rho u)}{\partial \theta} + \frac{\partial(\rho v)}{\partial z} = 0,$$
- where u , v are the velocity perpendicular and parallel to z .

Section - C

$2 \times 16 = 32$

(Long Answer Type Questions)

Note: Section 'C' contains 4 Long Answer Type Questions. Examinees will have to answer any two (02) questions. Each question is of 16 marks. Examinees have to delimit each answer in maximum 500 words.

- 10) A uniform vertical circular plate, of radius a ; is capable of revolving about a smooth horizontal axis through its centre; a rough perfectly flexible chain, whose mass is equal to that of the plate and whose length is equal to its circumference, hangs over its rim in equilibrium, if one end be slightly displaced, show that the velocity of chain, when the end reaches the plate is $\sqrt{\frac{\pi a g}{6}}$.
- 11) A uniform rod is placed with one end in contact with a horizontal table, and is then at an inclination α to the horizon and is allowed to fall. When it becomes horizontal, show that its angular velocity is $\sqrt{\frac{3g \sin \alpha}{2a}}$, whether the plane be perfectly smooth or perfectly rough. Show also that the end of the rod will not leave the plane in either case.

- 12) An infinite fluid in which is a spherical hollow shell of radius a is initially at rest under the action of no forces. If a constant pressure P is applied at infinity, show that the time of filling up the cavity is

$$\pi^2 a \left(\frac{\rho}{P} \right)^{1/2} \cdot 2^{5/6} \left(\sqrt{\frac{1}{3}} \right)^{-3}$$

- 13) A mass of liquid surrounds a solid sphere of radius a and its outer surface, which is a concentric spheres of radius b , is subject to a given constant pressure P , no other force being in action on the liquid. The solid sphere suddenly shrinks into a concentric sphere. Determine the subsequent motion and the impulsive action on the sphere.

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