MA/MSCMT-04

June - Examination 2018

M.A. / M.Sc. (Previous) Mathematics Examination

Differential Geometry and Tensors

Paper - MA/MSCMT-04

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C. Write answer as per the given instructions. Use of non-programmable scientific calculator is allowed in this paper.

Section - A **$8 \times 2 = 16$**

(Very Short Answer Type Questions)

Note: Section 'A' contains Very short Answer Type Questions. Examinees have to attempt all questions. Each question is of 02 marks and maximum word limit may be thirty words.

1) (i) Write the equation of osculating plane.

(ii) Define rectifying plane.

(iii) Define Bertrands curves.

(iv) Write down the equation of the evolute.

(v) Define lines of curvature.

(vi) Write the condition for two directions (du, dv) and (Du, Dv) to be conjugate.

(vii) State Gauss-Bonnet theorem.

(viii)Define conjugate metric tensor.

Section - B **$4 \times 8 = 32$**

(Short Answer Type Questions)

Note: Section 'B' contain 08 short Answer Type Questions. Examinees will have to answer any four (04) questions. Each question is of 08 marks. Examinees have to delimit each answer in maximum 200 words.

2) Find the inflexional tangents at (x, y, z) on the surface $y^2z = 4ax$

3) Prove that the principal normals at consecutive points of a curve do not intersect unless $\tau = 0$, where τ is torsion.

4) Find the equation to the edge of regression of the developable surface $y = xt = t^3 \ , z = t^3y - t^6$

5) Prove that the equation $Edu^2 - Gdv^2 = 0$ denote the curves bisecting the angles between the parametric curves u = constant, v = constant on a surface $\vec{r} = \vec{r}\left(u, v\right)$

6) Find the principal radii at the origin of the surface $2z = 5x^2 + 4xy + 2y^2$

7) Find the asymptotic lines on the surface $z = y \sin x$

8) If a metric of a V_3 is given by

$$ds^2 = 5(dx^1)^2 + 3(dx^2)^2 + 4(dx^3)^2 - 6(dx^1)(dx^2) + 4(dx^2)(dx^3)$$

Find g^{ij}

9) If surface of sphere is a two dimensional Riemannian space. Compute the Christoffel symbols.

Section - C **$2 \times 16 = 32$**

(Long Answer Type Questions)

Note: Section 'C' contains Four Long Answer Type Questions. Examinees will have to answer any two (02) questions. Each question is of 16 marks. Examinees have to delimit each answer in maximum 500 words.

10) It the radius of spherical curvature is constant show that the curve either lies on a sphere or has a constant curvature $R^2 = \rho^2 + (\sigma\rho')^2$, where R is a constant.

11) (i) Prove that the curvature and torsion of either associate Bertrand curves are connected by a linear relation.

(ii) Prove that the torsion of the two Bertrand curves have the same sign and their product is constant.

12) Derive the Canonical equations of a geodesic on the surface $\vec{r} = \vec{r}\left(u, v\right)$

13) State and prove Ricci's theorem.

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