MA/MSCMT-02

June - Examination 2018

M.A. / M.Sc. (Previous) Mathematics Examination

Real Analysis and Topology

Paper - MA/MSCMT-02

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C. Write answers as per the given instructions.

Section - A **8 × 2 = 16**

(Very Short Answer Type Questions)

Note: Section 'A' contain Eight (08) Very Short Answer Type Questions. Examinees have to attempt all questions. Each question is of 2 marks and maximum word limit may be thirty words.

1) (i) Define measurable set.

(ii) Define Lebesgue measure of a set.

(iii) Write Bolzano - Weierstrass property.

(iv) Write the necessary and sufficient conditions for a bounded function f defined on the interval $[a, b]$, to be L-integrable.

(v) State Riesz-Fisher theorem.

(vi) State Parseval's identity.

(vii) Define Kolomogorov space.

(viii)Define embedding.

Section - B **4 × 8 = 32**

(Short Answer Type Questions)

Note: Section 'B' contain Eight Short Answer Type Questions. Examinees will have to answer any four (04) questions. Each question is of 08 marks. Examinees have to delimit each answer in maximum 200 words.

2) Prove that the intersection of two measurable sets is also a measurable set.

3) Let f be a measurable function finite on $E = [a, b]$. Then prove that for given $\in\ > 0$, there exists a function φ, continuous on $[a, b]$ such that $m(\{x \in E : f(x) \neq \varphi(x)\}) < \in$.

4) If f is a bounded measurable function defined on a measurable set E, then prove that $|f|$ is L – integrable over E and

$$\left| \int_E f(x)\,dx \right| \leq \int_E |f(x)|\,dx$$

5) State and prove Minkowski's inequality.

6) Let A and B be subsets of a topological space (X, τ), then prove that

$$\overline{A \cup B} = \overline{A} \cup \overline{B}$$

7) Show that every subspace of T_2 – space is a T_2 – space.

8) Prove that every closed subset of locally compact space is locally compact.

9) Prove that a closed sub space of normal space is a normal space.

Section - C **$2 \times 16 = 32$**

(Long Answer Type Questions)

Note: Section ‘C’ contain 4 Long Answer Type Questions. Examinees will have to answer any two (02) questions. Each question is of 16 marks. Examinees have to delimit each answer in maximum 500 words.

10) State and prove Weierstrass approximation theorem.

11) Let $< f_n >$ be a sequence of bounded measurable functions defined on a set E of finite measure. If there exists a positive number M such that $|f_n(x)| \leq M$ for all $n \in N$ and for all $x \in E$ and if $< f_n >$ converges in measure to a bounded measurable function f on E, then prove that

$$\lim_{n \to \infty} \int_E f_n(x)\, dx = \int_E f(x)\, dx.$$

12) Prove that closure of a connected set is connected.

13) Prove that a series $\sum_{i=1}^{\infty} f_i$ of pair wise orthogonal elements in L_2 is converges iff the series of real numbers $\sum_{i=1}^{\infty} \| f_i \|^2$ is convergent.

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