MA/ MSCMT-01

June - Examination 2018

M.A./M.Sc. (Previous) Mathematics Examination

Advanced Algebra

Paper - MA/ MSCMT-01

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C. Write answers as per given instructions.

Section - A **$8 \times 2 = 16$**

(Very Short Answer Questions)

Note: Answer **all** questions. As per the nature of the question delimit your answer in one word, one sentence or maximum upto 30 words. Each question carries 2 marks.

1) (i) Define automorphism on a group.

(ii) Define derived subgroup of a group.

(iii) Define module homomorphism.

(iv) Define Algebraic field extension.

(v) Define unit element in a ring.

(vi) Define Galois group.

(vii) Define rank of a matrix.

(viii) Define inner product space.

Section - B **$4 \times 8 = 32$**

(Short Answer Questions)

Note: Answer **any four** questions. Each answer should not exceed 200 words. Each question carries 8 marks.

2) Suppose G_1 and G_2 are groups. Let $G = G_1 \times G_2$, $\hat{G}_1 = G_1 \times \{e_2\}$ and $\hat{G}_2 = \{e_1\} \times G_2$, then show that G is an internal direct product of $\hat{G}_1$ and $\hat{G}_2$.

3) If $a \in G$ then prove that two elements $x, y \in G$ give rise to the same conjugate of a if and only if they belong to the same right coset of $N(a)$ in G.

4) If G is a solvable group then prove that every subgroup of G is also solvable.

5) Let R be a Euclidian ring. Then prove that every non zero element in R can be written as the product of a finite number of primes of R or is a unit in R.

6) If $Q : M \to M'$ is an R module homomorphism: Then prove that ϕ is monomorphism if and only if $\text{Ker } \phi = \{0\}$

7) Let K/F be a field extension. Then show that G(K / F) of all automorphisms of K which leave every element of F fixed is a subgroup of the group Aut (K).

8) Let A be an nxn matrix over a field F. Then show a scaler $\lambda \in F$ is an eigen value of A if and only if $\det (A - \lambda I) = 0$

9) Show that an orthonormal set $\{V_1, V_2 .... V_n\}$ of non zero vectors in an innerproduct space V is linearly independent.

Section - C **$2 \times 16 = 32$**

(Long Answer Questions)

Note: Answer **any two** questions. You have to delimit your each answer maximum upto 500 words. Each question carries 16 marks.

10) State and prove Cayle's theorem.

11) Show that any two composition series of a group G are equivalent.

12) Let $t : V \rightarrow V'$ be a linear transformation and V is finite dimensional, then show that dim V = rank(t) + nullity (t)

13) Let F be a field and let P(x) be an arbitrary polynomial of positive degree over F. Then show that any two splitting field of P(x) are isomorphic. Also the isomorphic mapping can be so chosen that each element of F is mapped on to itself and the set of roots of P(x) in one splitting field is mapped one-one onto the set of roots of P(x) in mother splitting field.