MA/MSCMT-04

June - Examination 2017

M.A./M.Sc.(Previous) Mathematics Examination

Differential Geometry and Tensors

Paper - MA/MSCMT-04

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections. A, B and C. Write answer as per the given instructions. Use of non-programmable scientific calculator is allowed in this paper.

Section - A **$8 \times 2 = 16$**

(Very Short Answer Type Questions)

Note: Section 'A' contain 08 Very Short Answer Type Questions. Examinees have to attempt all questions. Each question is of 02 marks and maximum word limit may be thirty words.

1) (i) Define Inflexigonal tangent.

(ii) Define skew-curvature and give the formula for its magnitude.

(iii) Define singular points and give the formula of tangents at singular points.

(iv) Define developable surfaces.

(v) What is anchor ring.

(vi) Examine whether the parametric curves $x = b \sin u \cos v$; $y = b \sin u \sin v$; $z = b \cos u$, on a sphere of radius b constitute an orthogonal system.

(vii) Define mean normal curvature.

(viii)Define geodesic and give the differential equation of geodesic on a given surface.

Section - B **4 × 8 = 32**

(Short Answer Type Questions)

Note: Section 'B' contain 08 Short Answer Type Questions. Examinees will have to answer any four (04) questions. Each question is of 08 marks. Examinees have to delimit each answer in maximum 200 words.

2) Prove that $\frac{d\hat{t}}{ds} = k.\hat{n}.$

3) Find Weingarten equations for a given surface.

4) Let $v^2 du^2 + v^2 dv^2$ be the matric of a given surface, then find the matric corresponding to the new parameters so that these two families are parametric curves.

5) Find the value of first curvature and Gaussian curvature, at any point of right helicoid $x = u \cos\theta;\ y = u \sin\theta;\ z = c\theta$

6) Prove that necessary and sufficient condition that the parametric curves through a point to have conjugate directions is that M = 0.

7) For any surface, prove that $\frac{\partial(\log H)}{\partial u} = l + \mu;\ \frac{\partial(\log H)}{\partial v} = m + v.$

8) State and prove Ricci's theorem.

9) Prove that the unit tangent vectors form a field of parallel vectors along a geodesic.

Section - C **2 × 16 = 32**

(Long Answer Type Questions)

Note: Section 'C' contain Four Long Answer Type Questions. Examinees will have to answer any two (02) questions. Each question is of 16 marks. Examinees have to delimit each answer in maximum 500 words.

10) State and prove Gauss Bonnet theorem.

11) Show that the metric of a Euclidean space, referred to spherical coordinates is given by $ds^2 = (dr)^2 + (rd\theta)^2 + (r \sin\theta\, d\phi)^2$. Determine its metric tensor and conjugate metric tensor.

12) Find the equation of the developable surface whose generating line passes through the curve $y^2 = 4ax, z = 0; x^2 = 4ay, z = c$, and show that its edge of regression is given by,

$cx^2 - 3ayz = 0 = cy^2 - 3ax(c - z)$

13) (i) How to contract Riemann Christoffel tensor to Ricci tensor.

(ii) Prove that the divergence of the Einstein tensor vanishes.