MA/MSCMT-03

June - Examination 2017

M.A./M.Sc.(Previous) Mathematics Examination

Differential Equations, Calculus of Variations and Special Functions

Paper - MA/MSCMT-03

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections. A, B and C. Write answer as per the given instructions. Use of non-programmable scientific calculator is allowed in this paper.

Section - A **$8 \times 2 = 16$**

(Very Short Answer Type Questions)

Note: Section 'A' contain 08 Very Short Answer Type Questions. Examinees have to attempt all questions. Each question is of 02 marks and maximum word limit may be thirty words.

1) (i) Write general form of the Riccati's equation.

(ii) Write the necessary and sufficient condition for the total differential equation $Pdx + Qdy + Rdz = 0$ to be integrable.

(iii) If $I = \int_a^b f(x, y, y')\,dx$ and f is independent of y', then write solution of Euler-Lagrange equation.

(iv) Write three dimensional Laplace equation in polar coordinate system.

(v) Classify the following partial differential equation:

$$3\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + 5\frac{\partial^2 z}{\partial y^2} + x\frac{\partial z}{\partial y} = 0.$$

(vi) Define functional.

(vii) Write generating function for Bessel function.

(viii)Define associated Laguerre polynomial.

Section - B $\quad 4 \times 8 = 32$

(Short Answer Type Questions)

Note: Section 'B' contain 08 Short Answer Type Questions. Examinees will have to answer any four (04) questions. Each question is of 08 marks. Examinees have to delimit each answer in maximum 200 words.

2) Solve:

$$\frac{d^2 y}{dx^2} + 2\frac{dy}{dx} + 4\left(\frac{dy}{dx}\right)^3 = 0.$$

3) Solve by the method of separation of variables the PDE

$$4\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 3u, \text{ given that } u = 3e^{-x} - 3^{5x} \text{ when } t=0$$

4) Find the eigenvalues and eigenfunctions for the boundary value problem

$$y'' - 2y + \lambda y = 0; \; y(0) = 0, \; y(\pi) = 0.$$

5) Find extremals of the functional

$$F[y(x)] = \int_0^{\pi/2} (y'^2 - y^2)\,dx,\ \ y(0) = 0,\ \ y(\pi/2) = 1.$$

6) Prove that the eigenvalues of Sturm-Liouville system are real.

7) Prove that

$$\frac{d^m}{dx^m}[x^{a-1+m}{}_2F_1(a, b; c; x)] = (a)_m x^{a-1}{}_2F_1(a+m, b; c; x).$$

8) Show that

$$(1 - 2xh + h^2)^{-1/2} = \sum_{n=0}^{\infty} P_n(x)h^n.|x| \le 1,|h| \le 1.$$

9) State and prove Rodrigue's formula for $L_n(x)$. (Laguerre polynomial).

Section - C **2 × 16 = 32**

(Long Answer Type Questions)

Note: Section 'C' contain Four Long Answer Type Questions. Examinees will have to answer any two (02) questions. Each question is of 16 marks. Examinees have to delimit each answer in maximum 500 words.

10) Solve $r + (a+b)s + abt = xy$ by Monge's method.

11) State and prove Euler-Lagrange's equation.

12) Find series solution for the differential equation:

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - 1)y = 0$$

13) State and prove orthogonal property for Legendre polynomial.
