MA/MSCMT-10

June - Examination 2017

M.A./M.Sc. (Final) Mathematics Examination

Mathematical Programming

Paper - MA/MSCMT-10

Time : 3 Hours]   **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C. Write answer as per the given instructions.

Section - A   **8 × 2 = 16**
Very Short Answer Questions

Note: Section 'A' contain 08 Very Short Answer Type Questions. Examinees have to attempt all questions. Each question is of 02 marks and maximum word limit may be thirty words.

1) (i) State the relation between quadratic form and convex function.

    (ii) Define Slack and surplus variable.

    (iii) Write the condition when a point will be saddle point in Lagrangian function.

    (iv) Define quadratic programming problem.

    (v) Define separable functions.

    (vi) Define transition function in Dynamic programming.

(vii) Determine whether or not, the quadratic forms $A^T AX$ are positive definite, where $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

(viii) Prove that if (X_0, λ_0) is a saddle point of the function $F(X, \lambda)$ for every $\lambda \geq 0$, then X_0 is a minimal point of $f(X)$ subject to constraints $G(X) \leq 0$.

Section - B $4 \times 8 = 32$
Short Answer Questions

Note: Section 'B' contain 08 Short Answer Type Questions. Examinees will have to answer any four (04) questions. Each question is of 08 marks. Examinees have to delimit each answer in maximum 200 words.

2) Prove that $f(x) = \dfrac{1}{x}$ is strictly convex for $x > 0$ and strictly concave for $x < 0$.

3) Obtain necessary conditions for the optimum solution of the following non linear programming problem:

Min $z = f(x_1, x_2) = 3e^{2x_1 + 1} + 2e^{2x_2 + 5}$ subject to constraints $x_1 + x_2 = 7$ and $x_1, x_2 \geq 0$.

4) Use Lagrangian function to find the optimal solution for the following non linear problem:

Maximize $f(X) = -3x_1^2 - 4x_2^2 - 5x_3^2$ subject to

$x_1 + x_2 + x_3 = 10; x_1, x_2, x_3 \geq 0$.

5) Write the Kuhn Tucker necessary and sufficient conditions for the following nonlinear programming problem to have on optimal solution: Max $f(x_1, x_2) = x_1^2 - 2x_1 - x_2$ s.t.

$2x_1 + 3x_2 \leq 6; 2x_1 + x_2 \leq 4; x_1, x_2 \geq 0$.

6) Give the basic steps involved in duality in quadratic programming.

7) Make use of dynamic programming, show that $\sum_{i=1}^{n} p_i \log pi$ subject to $\sum_{i=1}^{n} pi = 1; pi > 0$ is minimum when $p_1 = p_2 = p_3 = \cdots = p_n = \dfrac{1}{n}$.

8) Use dynamic programming to solve the following LPP: Max $z = 2x_1 + 5x_2$ s.t.

$2x_1 + x_2 \leq 43; 2x_2 \leq 46; x_1, x_2 \geq 0$.

9) Prove that every local maximum of the general convex programming problem is its global maximum.

Section - C   **2 × 16 = 32**

Long Answer Questions

Note: Section 'C' contain 04 Long Answer Type Questions. Examinees will have to answer any two (02) questions. Each question is of 16 marks. Examinees have to delimit each answer in maximum 500 words. Use of non-programmable scientific calculator is allowed in this paper.

10) Using bounded variation technique, solve the following lpp
    Max $z = x_1 + 3x_2$ s.t. $x_1 + x_2 + x_3 \leq 10$; $x_1 - 2x_3 \geq 0$;
    $2x_2 - x_3 \leq 10$; $0 \leq x_1 \leq 8, 0 \leq x_2 \leq 4, x_3 \geq 0$.

11) Solve the following quadratic programming using Wolfe's method:
    Minimize $f(x_1, x_2) = x_1^2 - x_1 x_2 + 2x_2^2 - x_1 - x_2$ s.t.
    $2x_1 + x_2 \leq 1$; $x_1, x_2 \geq 0$.

12) A manufacturer of baby doll makes two types of dolls doll x and doll y. Processing of these two dolls is done on two machine, A and B. Doll x requires 2 hours on machine A and 6 hours on machine B. Doll y requires 5 hours on machine A and also 5 hours on machine B. There are sixteen hours of time per day available on machine A and thirty hours on machine B. The profit gained on both the dolls is same, i.e. one rupee per doll. What should be the daily production of the two dolls for maximum profit?
    (i) Set up and solve the lpp.
    (ii) If optimal solution is not integer valued, use Gomory's technique to derive the optimal solution.

13) Solve the following lpp by Branch and Bound technique Max $z = 7x_1 + 9x_2$ s.t. $-x_1 + 3x_2 \leq 6$; $7x_1 + x_2 \leq 35$; $x_1 x_2 \geq 0$ and x_1, x_2 are integers.

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