MA/MSCMT-09

June - Examination 2017

M.A./M.Sc. (Final) Mathematics Examination

Integral Transforms and Integral Equations

Paper - MA/MSCMT-09

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C. Write answers as per the given instructions. USe of non-programmable scientific calculator is allowed in this paper.

Section - A **$8 \times 2 = 16$**

(Very Short Answer Questions)

Note: Answer all questions. As per the nature of the question should be given in 30 words. Each question carries 2 marks

1) (i) State existence condition of Laplace Transform.

(ii) Find Laplace transform of Dirac Delta function.

(iii) Write the relationship between Fourier transform and Laplace transform.

(iv) State the Parseval's Identity for Fourier cosine transform.

(v) Showthat $M\{\log x\ f(x); p\} = \frac{d}{dp} F(p), if\ M\{f(x); p\} = F(p)$.

(vi) Write definition of Hankel transform.

(vii) If we want to remove the term $\frac{\partial^2 U}{\partial x^2}$ from a PDE then at $x = 0$, what type of condition is required in the case of (i) Fourier cosine transform and (ii) Fourier sine transform.

(viii) Define the Integral Equation of Convolution type.

Section - B $4 \times 8 = 32$

(Short Answer Questions)

Note: Answer any four questions. Each answer should be given in 200 words. Each question carries 8 marks.

2) Given that $L\{\sin\sqrt{t}\} = \frac{\sqrt{\pi}}{2p^{3/2}} e^{-1/4p}$, determine the Laplace transform of $\frac{\cos\sqrt{t}}{\sqrt{t}}$

3) Evaluate $L^{-1}\left[\frac{1}{(P^2 + 2P + 5)^{3/2}}; t\right]$

4) Find the Fourier transform of the function e^{-at^2}, $a > 0$.

5) If $M\{f(x); p\} = F(p)$, then show that

$$M\left\{\int_0^x dy \int_0^y f(u)\, du; p\right\} = \frac{1}{p(p+1)} F(p+2)$$

6) Find the Hankel transform of the function.

$$f(x) = \begin{cases} x^n, 0 < x < a \\ 0, \quad x > a, \qquad n > -1 \end{cases}$$

taking $xJ_n(px)$ as the Kernel.

7) Show that the function $g(x) = 1$ is a solution of the Fredholm integral equation $g(x) + \int_0^1 x(e^{xt} - 1)\, g(t)\, dt = e^x - x$.

8) Solve the integral equation

$$g(x) = e^{-x} - 2\int_0^x \cos(x-t)\,g(t)\,dt.$$

9) Find the resolvent kernel of the kernel

$$K(x, t) = e^{x+t}, \text{ with } a = 0,\ b = 1$$

Section - C **2 × 16 = 32**

(Long Answer Questions)

Note: Answer any two questions. You have to delimit your each answer maximum up to 500 words. Each question carries 16 marks.

10) Solve the following differential equation by using Laplace transform:

$$y'' + 5y' + 6y = 1 - u_3(t) - u_5(t);\ y(0) = y'(0) = 0$$

where $u_a(t) = H(t-a)$ is a Unit-step/Heaviside function.

11) Find f (t), if its Fourier sine transform is $\frac{p}{1+p^2}$.

12) Find the eigenvalues and eigenfunction of the homogeneous integral equation.

$$g(x) = \lambda\int_0^{\pi} [\cos^2 x \cos 2t + \cos 3x \cos^3 t]\, g(t)\,dt.$$

13) Solve the symmetric integral equation.

$$g(x) = (x+1)^2 + \int_{-1}^{1} (xt + x^2 t^2)\, g(t)\,dt$$

by using Hilbert-Schmidt theorem.