MA/MSCMT-06

June - Examination 2017

M.A./ M.Sc. (Final) Mathematics Examination

Analysis and Advanced Calculus

Paper - MA/MSCMT-06

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C. Write answer as per the given instructions.

Section - A $8 \times 2 = 16$

Very Short Answer Questions

Note: Section 'A' contain 8 very short answer type questions. Examinees have to attempt all questions each question is of 2 marks and maximum word limit may be thirty words.

1) (i) Define Convergence in Normed Linear space.

(ii) Define Closed linear transformation.

(iii) Define Inner product.

(iv) What is step function for a Banach Space.

(v) Define Directional derivative in Banach Space.

(vi) Define Perpendicular Projection for a Hilbert Space.

(vii) Define Orthogonal Complement.

(viii) State Global Uniqueness Theorem for a Banach Space.

Section - B **4 × 8 = 32**

Short Answer Questions

Note: Section 'B' contain 8 short answer type questions. Examinees will have to answer any 4 questions, each question is of 8 marks. Examinees have to delimit each answer in maximum 200 words.

2) Show that for a finite dimensional linear space, all norms are equivalent.

3) If x and y are any two vectors in a Hilbert space H. Then show that

(i) $\|x+y\|^2 - \|x-y\|^2 = 4\,\mathrm{Re}(x, y)$.

(ii) $(x, y) = \mathrm{Re}(x, y) + i\,\mathrm{Re}(x, iy)$

4) If T is an operator on a Hilbert space H, Then T is normal iff its real and imaginary parts commute.

5) If P and Q are projections on closed linear spaces M and N of a Hilbert space H, then $\mathrm{M} \perp \mathrm{N} \Leftrightarrow \mathrm{PQ} = 0 \Leftrightarrow \mathrm{QP} = 0$

6) Let f be a regulated function on a compact interval $[a, b]$ of R into a Banach space X over K and g be a continuous linear map of X into a Banach space Y over K. Then gof is regulated and $\int_a^b gof = g\left(\int_a^b f\right)$.

7) Let u be a non-negative continuous function on an interval $\{0, c\}$, $(c > 0)$ satisfying the inequality

$u(t) \le at + k\int_0^t u(s)\,ds$ for all $t \in [0, c]$ then $u(t) \le \frac{a}{k}(e^{kt} - 1)$

for $t \in [0, c]$

8) Show that every convergent sequency in Normed linear space is a cauchy sequence but it converse need not be true.

9) State and prove open mapping theorem.

Section - C **2 × 16 = 32**

Long Answer Questions

Note: Section 'C' contain 4 short answer type questions. Examinees will have to answer any 2 questions, each question is of 16 marks. Examinees have to delimit each answer in maximum 500 words.

10) Let N be an arbitrary normed linear space. Then for each vector x in N induces a functional F_x on N^{**} defined by

$$F_x(f) = f(x), \forall f \in N^{**} \ s.t. \|F_x\| = \|x\|$$

Further prove that the mapping $J : N \to N^{**}$ defined as

$J(x) = F_x$, $\forall x \in N$ is an isometric isomorphism of N into N^{**}.

11) State and prove Taylor's formula with Lagrange's remainder for differentiable function over Banach space.

12) State and prove Bessel's inequality for finite orthonormal sets.

13) If T be a linear transformation of Normed linear space N into a normed linear space N', then prove that inverse of T i.e. T^{-1} exists and is continuous on its domain of definition iff $\exists$ a constant $K \geq 0$ s.t. $K\|x\| \leq \|T(x)\|, \forall x \in N$.