MA/MScMT-05

June - Examination 2017

M.A. / MSc. (Previous) Mathematics Examination

Mechanics

Paper - MA/MScMT-05

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C.

Section - A **$8 \times 2 = 16$**

(Very Short Answer Type Questions)

Note: Examinees have to attempt all questions. Each question is of 02 marks and maximum word limit may be thirty words.

1) (i) Define moving axes and fixed axes.

(ii) Write Euler's Geometrical equation of motion.

(iii) Write the principle of conservation of Linear momentum.

(iv) Define the generalized co-ordinates.

(v) Write the Hamilton's principle.

(vi) Define the pressure.

(vii) Define a stagnation point.

(viii) Define a doublet.

Section - B **4 × 8 = 32**

(Short Answer Type Questions)

Note: Examinees will have to answer any four (04) questions. Each question is of 08 marks. Examinees have to delimit each answer in maximum 200 words.

2) Prove that the centre of suspension and centre of oscillation are convertible (or interchangeable.)

3) A uniform solid cylinder is placed with its axis horizontal on a plane. Whose inclination to the horizon is α. Show that the least coefficient of friction between it and the plane, so that is may roll and not slide is $\frac{1}{3}\tan\alpha$.

4) A body having an axis of symmetry OC, moves about a fixed point O under no forces except a constant retarding couple kC about the axis OC. If A, A, C are the moments of inertia and w_1, w_2, w_3 the angular velocities about the principal axes OA, OB, OC, Show that at time t,

$$w_1 = \Omega\cos\left[\lambda t\left(\Omega - \frac{1}{2}kt\right)\right],\ w_2 = -\,\Omega\sin\left[\lambda t\left(\Omega - \frac{1}{2}kt\right)\right],$$

$w_3 = \Omega - \text{kt}$ where $\lambda = \dfrac{A - C}{A}$, the initial values of w_1, w_2, w_3 being Ω, 0, Ω respectively.

5) Show that for a body of revolution the maximum value of the angle between the axis of the impulsive couple acting on it and the instantaneous axis of initial motion set up by the couple in the body if $\sin^{-1}\left(\dfrac{C - A}{C + A}\right)$.

6) A circular plate is turning in its own plane about a point A on its circumference. Suddenly A is freed and point B, also on the circumference, fixed. Show that the plate will be reduced to rest if the arc AB is one third of the circumference.

7) Deduce the principle of energy from the Lagrange's equations.

8) Given $u = Wy, v = Wx \; and \; w = 0$, show that the surfaces intersecting the stream line orthogonally exist and are the planes through z-axis.

9) What arrangement of sources and sinks will give rise to the function.

$w = \log\left(z - \frac{a^2}{z}\right)$?

Draw a rough sketch of a stream line. Prove that two of the stream lines sub divide into the circle r = a and the axis of y.

Section - C **2 × 16 = 32**

(Long Answer Type Questions)

Note: Examinees will have to answer any two (02) questions. Each question is of 16 marks. Examinees have to delimit each answer in maximum 500 words. Use of non-programmable scientific calculator is allowed in this paper.

10) A rod of length 2a is suspended by a string of length l, attached to one end; if the string and rod revolve about the vertical with uniform angular velocity and their inclinations to the vertical be θ and ϕ respectively, show that:

$$\frac{3l}{a} = \frac{(4\tan\theta - 3\tan\phi)\sin\phi}{(\tan\phi - \tan\theta)\sin\theta}$$

11) If initially the axis of the top is horizontal and it is set spinning with angular velocity ω in a horizontal plane, prove that the axis will start to rise if $nC\omega > mgh$ and that when $nC\omega = 2mgh$ the axis will rise to an angular distance $\cos^{-1}\left(\frac{A\omega}{nC}\right)$, provided that $A\omega < nC$ and will there be at instantaneous rest A, C and n have their usual meanings.

12) Obtain the 'Cauchy's Integrals' equations for motion of top.

13) If the lines of motion are curves on the surface of spheres all touching the plane of xy at the origin O, then prove that the equation of continuity is

$$r\sin\theta\,\frac{\partial p}{\partial t}+\frac{\partial(pv)}{\partial\phi}+\sin\theta\,\frac{\partial(pu)}{\partial\theta}+pu\,(1+2\cos\theta)=0$$
