MA/MSCMT-02

June - Examination 2017

M.A. / M.Sc. (Previous) Mathematics Examination

Real Analysis and Topology

Paper - MA/MSCMT-02

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C.

Section - A **8 × 2 = 16**

(Contain 08 Very Short Answer Type Questions)

Note: Examinees have to attempt all questions. Each question is of 02 marks and maximum word limit may be thirty words.

1) (i) Define measurable set.

(ii) Define directed set.

(iii) Define orthonormal system.

(iv) State Riesz-Fisher theorem.

(v) State Holder's inequality.

(vi) State Parseval's identity

(vii) Define Topological space.

(viii) Define embedding.

Section - B **$4 \times 8 = 32$**

(Contain 08 Short Answer Type Questions)

Note: Examinees will have to answer any four (4) question. Each question is of 08 marks. Examinees have to delimit each answer in maximum 200 words.

2) Prove that the union of two measurable sets is a measurable set.

3) Let f be a measurable function define of $E = [a, b]$. Then prove that for given $\in > 0$, there exists a function φ, continuous on $[a, b]$ such that $m(\{ x \in E : f(x) \neq \varphi(x)\}) < \in$.

4) Show that every bounded measurable function f defined on a measurable set E is L-integrable.

5) State and prove Minkowski's inequality.

6) Prove that homeomorphism is an equivalence relation in the family of topological spaces.

7) Show that regularity is a topological property.

8) Prove that T_∞ is a topology on X_∞.

9) Prove that the product space $(X \times Y, W)$ is compact if and only if each of the spaces (X, τ) and (X, V) is compact.

Section - C **$2 \times 16 = 32$**

(Contain 4 Long Answer Type Questions)

Note: Examinees will have to answer any two (02) questions. Each question is of 16 marks. Examinees have to delimit each answer in maximum 500 words. Use of non-programmable scientific calculator is allowed in this paper.

10) State and prove Weierstrass approximation theorem.

11) Let $< f_n >$ be a sequence of bounded measurable functions defined on a set E of finite measure. If there exists a positive number M such that $| f_n(x) | \leq M$ for all $n \in N$ and for all $x \in E$ and if $< f_n >$ converges in measure to a bounded measurable function f on E, then prove that $\lim_{n \to \infty} \int_E f_n (x)\, dx = \int_E f (x)\, dx.$

12) (i) If E is a countable set, then show that $m^*(E) = 0$

(ii) Prove that a closed subset of a compact space is compact.

13) Prove that a series $\sum_{i=1}^{\infty} fi$ of pair wise orthogonal elements in L_2 is converges iff the series of real numbers $\sum_{i=1}^{\infty} \| fi \|^2$ is convergent.

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