MA/MSCMT-01

June - Examination 2017

M.A./M.Sc.(Previous)Mathematics Examination

Advanced Algebra

Paper - MA/MSCMT-01

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C. Write answer as per the given instructions.

Section - A $8 \times 2 = 16$

Very Short Answer Questions

Note: Section 'A' contains 08 Very Short Answer type questions examinees have to attempt all questions. Each questions is of 02 marks and maximum word limit may be thirty words.

1) (i) Define external direct product of groups.

(ii) Define conjugate class.

(iii) Define derived sub group.

(iv) Define unit in a ring.

(v) Define module monomorphism.

(vi) Define minimal polynomial of an algebraic element.

(vii) Define fixed field of a group of automorphism.

(viii) Define rank of a matrix.

Section - B **$4 \times 8 = 32$**

Short Answer Questions

Note: Section 'B' contain 08 short Answer type Question Examinees will have to answer any four (04) questions. Each questions is of 08 marks. Examines have to delimit each answer in maximum 200 words.

2) Prove that a subgroup N of the group G is normal if and only if it is the kernel of some homomorphism.

3) Let $a \in G$. Then prove that two elements $x,\ y \in G$ give rise to the same conjugate of a if and only if they belong to the same right coset of N(a) in G, where N(a) is normalizer of a in G.

4) If G is a solvable group, then prove that every homomorphic image and every quotient group of G is also solvable.

5) Let M be an R-module and $N_1, N_2, ..., N_k$ be sub module of M. Then prove that $M = N_1 \oplus N_2 \oplus ..... N_k$

if and only if

(i) $M = N_1 + N_2 + ... + N_k$ and

(ii) $Ni \cap (N_1 + N_2 + ... + N_{i-1} + N_{l+1} + ... N_k) = \{0\}$,
for all $i = 1, 2, ..., k$

6) Prove that the order of the Galo is group G (K/F) is equal to the degree of K over F, that is $0[\,G(K/F)] = [K:F]$

7) If $t: V \rightarrow V'$ is an isomorphism, then prove that $\{V_1, V_2, ..., V_n\}$ is linear independent if and only if $\{(V_1), t(V_2), ..., t(V_n)\}$ is linearly independent where $v_i \in V ; 1 \leq i \leq n$.

8) Let V be an inner product space and $A = \{V_i\}_{i=1}^{n}$ be an orthonormal set in V. Then prove that for any vector $v \in V$, the vector $u = v - \sum_{i-1}^{n} \langle v_1 v_i \rangle v_i$ is orthogonal to each v_j, $j = 1, 2, ..., n$.

9) Let V be a unitary space. Then prove that for any arbitary vectors $u, v \in V$

$1\langle u, v \rangle 1 \leq || u || \ \ || v ||$.

Section - C **2 × 16 = 32**

Long Answer Questions

Note: Section 'C' contains 04 Long Answer Type Questions. Examinee will have to answer any two (2) questions. Each question is of 16 marks. Examinee have the delimit each answer in 500 words.

10) Let ϕ be a homomorphism of G onto G^1 with Kernal K. For N^1 a subgroup of G^1, let N be defined by $N = \{x \in G \mid \phi(x) \in N^1\}$. Then prove that N is a subgroup of G and $N \supset K$. Also prove that if N^1 is normal in G^1 then N is normal in G and $G/N \cong G^1/N^1$.

11) (i) Let R be a Enclidean ring and a and b be any non zero elements in R. If b is not a unit in R, then prove that $d(a) < d(ab)$.

(ii) If K/F is a finite extension, then prove that it is an algebraic extension.

12) Let F be a field and let $p(x)$ be an arbitrary polynomial of positive degreee over F. Then prove that any two splitting fields of $p(x)$ are isomorphic. Also prove that isomorphic mapping can be chosen that each element of F is mapped onto itself and set of roots of $p(x)$ in one splitting field is mapped one-one onto the set of roots of $p(x)$ in another splitting field.

13) Let V be on n - dimensional vector space over a field F with basis B and V^1 be an m - dimensional vector space over F with basis B^1. Then prove that the map Hom $(V, V^1) \rightarrow M_{m \times n}(F)$ from the space of linear transformation from V to V^1 to the space of $m \times n$ matrices with coefficients in F defined by $t \rightarrow M_{B^1}^{B}$ is an isomorphism that is

Horn $(V, V^1) \cong M_{m \times n}(F)$.

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