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MA/MSCMT-01

June - Examination 2016

M.A./M.Sc. (Previous) Mathematics Examination

Advanced Algebra

Paper - MA/MSCMT-01

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C. Write answer as per the given instruction. Use of non-programmable scientific calculator is allowed in this paper.

Section - A **$8 \times 2 = 16$**

(Very Short Answer Type Questions)

Note: Section 'A' contain 08 Very Short Answer Type Questions. Examinees have to attempt all questions. Each question is of 02 marks and maximum word limit may be thirty words.

1) (i) Define internal direct product.

(ii) Define solvable group.

(iii) Define quotient module.

(iv) Define prime elements.

(v) Define solvability by radicals.

(vi) Define ergen value.

(vii) Write Schwartz's inequality.

(viii) Define adjoint of a linear map.

Section - B $4 \times 8 = 32$

(Short Answer Type Questions)

Note: Section 'B' contain 08 Short Answer Type Questions. Examinees have to answer any four (4) questions. Each question is of 08 marks. Examinees have to delimit each answer in maximum 200 words.

2) Prove that a group G is abelian if and only if $G' = \{e\}$; e is identify of G.

3) Let R be a Euclidean ring and a and b be any two non zero elements in R. If b is not a unit in R, then prove that d (a) < d (ab)

4) If $\phi : M \rightarrow M'$ is an R-module homomorphism. Prove that ϕ is a monomorphism if and only if ker $\phi = \{0\}$

5) Prove that an irreducible polynomial $f(x)$ over a field F of characteristics $p > 0$ is inseparable if and only if $f(x)$ is a polynomial in x^p.

6) If v is a finite dimensional vector space over a field F, Prove that for every non-zero vector $v \in V$ there exists a linear functional f in V* s.t. $f(v) \neq 0$.

7) Let v, w, u be vector spaces over a field F. Let $\{v_j\}_{i=1}^{n}, \{w_i\}_{i=1}^{m}$ and $\{u_r\}_{r=1}^{k}$ be bases for v, w and u respectively. If $t : V \rightarrow W$, $s : W \rightarrow U$ are linear transformations, A and B are the matrices associated with t and s respectively. Then prove that the matrix associated with $(s.t)$ is BA.

8) If a square matrix A of order n, over a field f has n distinct ergen values $\lambda_1, \lambda_2, \ldots\ldots, \lambda_n$. Prove that there is an invertible matrix P such that $p^{-1} AP = \text{drag } (\lambda_1, \lambda_2, \ldots\ldots, \lambda_n)$.

9) Let v and v' be inner product spaces. Prove that a linear transformation $t : v \to v'$ is orthogonal if and only if $\| t(u) \| = \| u \|$ for all $u \in v$.

Section - C $2 \times 16 = 32$

(Long Answer Questions)

Note: Section 'C' contain 04 Long Answer Type Questions. Examinees will have to answer any two (2) questions. Each question is of 16 marks. Examinees have to delimit each answer in maximum 500 words.

10) (i) Prove that every group is isomorphic to a group of permutations.

(ii) Let G_1 and G_2 be groups. If N_i is normal in G_i; i = 1, 2. Prove that $N_1 \times N_2$ is normal in $G_1 \times G_2$ and $(G_1 \times G_2) / (N_1 \times N_2) \cong (G_1 \times N_1) \times (G_2 \times N_2)$

11) (i) Let k be the finite algebraic extension of a field F. Prove that k is a normal extension of F if and only if k is the splitting field of some polynomial over F.

(ii) If k is finite, separable and normal extension of a field F. Prove that an element of k which remain invariant for each member of Galo is group G(K/F) is necessarity a member of F.

12) Let V and V′ be finite dimensional vector spaces over a field F with bases B and B′ respectively. If $t : V \to V'$ is a linear transformation then prove that $M_{B^*}^{B'^*}(t^*) = [M_{B'}^{B}(t)]^T$ where t* is the dual map of t, B* and B′* are the dual bases of B and B′ respectively.

13) Prove that every finite dimensional vector space V with an inner product has an orthonormal basis.