MA/MSCMT-05

June - Examination 2016

M.A. / MSc. (Previous) Mathematics Examination

Mechanics

Paper - MA/MSCMT-05

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C. Write answer as per the given instructions.

Section - A **$8 \times 2 = 16$**

(Very Short Answer Questions)

Note: Section 'A' contain 08 very short Answer type Questions. Examinees have to attempt all questions. Each question is of 02 marks and maximum word limit may be thirty words.

1) (i) State D' Alembert's Principle.

(ii) In polar coordinates (r, θ), when the particle is moving along a circle of radius a, then give expressions for redial and transverse accelerations.

(iii) Write expression for K.E. of a rigid body in a two dimensional motion under finite forces.

(iv) Write Euler's equations of motion.

(v) Define invariable line.

(vi) What do you mean by conservation forces?

(vii) Write the equations of motion of a top.

(viii) Define stream function.

Section - B **$4 \times 8 = 32$**

(Short Answer Questions)

Note: Section 'B' contain 08 short answer type questions. Examinees will have to answer any four (04) questions. Each question is of 08 marks. Examinees have to delimit each answer in maximum 200 words.

2) State and prove D' Alembert's Principle.

3) A uniform thin circular disc is set rotating with an angular velocity w about an axis through the centre making an angle i with the normal. Prove that the semi-vertical angle θ of the core described by the axis of disc is given by tan $\tan\theta = \frac{1}{2}\tan i$.

4) A small insect moves along a uniform bar of mass equal to itself and of length $2a$, the ends of which are constrained to remain on the circumference of a fixed circle whose radius is $\frac{2a}{\sqrt{3}}$. If the insect starts from the middle point of the bar and move along the bar with relative velocity V, show that the bar in time t will turn through an angle $\frac{1}{\sqrt{3}}\tan^{-1}\frac{Vt}{a}$.

5) Drive the equation of motion of a simple pendulum by using Lagrange's equations.

6) The velocity components for a two dimensional flow system can be given in the Eulerian system by $u = 2x + 2y + 3t,\ v = x + y + \frac{1}{2}t$ Find the displacement of a fluid particle in the Lagrangian system.

7) Derive the equation of continuity by Euler's method.

8) State and prove Bernoulli's theorem.

9) Show that the velocity potential

$$\phi = \frac{1}{2}\log\frac{(x+a)^2+y^2}{(x-a)^2+y^2}$$

gives a possible motion. Determine the stream lines.

Section - C **2 × 16 = 32**

(Long Answer Questions)

Note: Section 'C' contain 04 long answer type questions. Examinees will have to answer any two (02) questions. Each question is of 16 marks. Examinees have to delimit each answer in maximum 500 words. Use of non-programmable scientific calculator is allowed in this paper.

10) Define the doublet and derive the complex potential for a doublet. Also derive the image of a doublet with respect to a circle.

11) Write down the condition for a surface representing a boundary surface. Show that the ellipsoid

$$\frac{x^2}{a^2k^2t^{2n}} + kt^n\left(\frac{y^2}{b^2}+\frac{z^2}{c^2}\right)=1$$

is a possible form of the boundary surface of a liquid.

12) A rectangular parallelepiped whose edges are a, $2a$, $3a$ can turn freely about its centre and is set rotating about a line perpendicular to the mean axis and making an angle $\cos^{-1}\dfrac{5}{8}$ with the least axis. Prove that ultimately the body will rotate mean axis.

13) A uniform rod is placed with one end in contact with a horizontal table, and is then at an inclination α to the horizon and is allowed to fall. When it becomes horizontal, show that its angular velocity is $\sqrt{\dfrac{3g\sin\alpha}{2a}}$, whether the plane be perfectly smooth or perfectly rough. Show also that the end of the rod will not leave the plane in either case.

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