MA/MSCMT-03

June - Examination 2016

M.A./M.Sc. (Previous) Mathematics Examination

Differential Equations, Calculus of Variations and Special Functions

Paper - MA/MSCMT-03

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C. Use of non-programmable scientific calculator is allowed in this paper.

Section - A **8 × 2 = 16**

(Very Short Answer Questions)

Note: Section 'A' contain (08) Very Short Answer Type Questions. Examinees have to attempt **all** questions. Each question is of 02 marks and maximum word limit may be thirty words.

1) (i) Write Rodrogue's formula for the Laguerre polynomial.

(ii) Write general form of the Riccati's equation.

(iii) Define an isoperimetric problem.

(iv) Write three dimensional wave equation in Cartesian coordinate system.

(v) Find the nature of following PDE:

$$3\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + 5\frac{\partial^2 z}{\partial y^2} + x\frac{\partial z}{\partial y} = 0$$

(vi) Write Euler-Lagrange equation for the stationary value of integral:

$$I = \int_{x_1}^{x_2} f\,(x, y, y', y'', y''')\,dx$$

(vii) Write Gauss's Hypergeometric differential equation.

(viii) Write orthogonal property for Legendre polynomial.

Section - B   4 × 8 = 32

(Short Answer Questions)

Note: Section 'B' contain Short Answer Type Questions. Examinees have to answer **any four** (04) questions. Each question is of 08 marks. Examinees have to delimit each answer in maximum 200 words.

2) Solve:

$$y\,(1 - \log y)\frac{d^2 y}{dx^2} + (1 + \log y)\left(\frac{dy}{dx}\right)^2 = 0$$

3) Using the method of separation of variables, solve the two dimensional heat conduction equation.

4) Find series solution of the Legendre's equation.

$$\left(1 - x^2\right)\frac{d^2 y}{dx^2} - 2x\frac{dy}{dx} + n\,(n+1)\,y = 0$$

5) Find externals of the functional.

$$\int_{x_1}^{x_2} \frac{\left(1+y'^2\right)^{1/2}}{x}\, dx$$

6) Define Gauss's Hypergeometric series and discuss its convergence conditions.

7) Prove that

$$B(\lambda, c-\lambda)\,{}_2F_1(a,b;c;z) = \int_0^1 t^{\lambda-1}(1-t)^{c-\lambda-1}\,{}_2F_1(a,b;c;zt)\,dt,$$

where $|z|<1, \lambda>0, c-\lambda>0$.

8) Show that

$$\exp\left\{\frac{x}{2}\left(z-\frac{1}{z}\right)\right\} = \sum_{n=-\infty}^{\infty} J_n(x) z^n$$

9) Prove that

$$\int_0^\infty e^{-st} L_n(t)\, dt = \frac{1}{s}\left(1-\frac{1}{s}\right)^n$$

Section - C **2 × 16 = 32**

(Long Answer Questions)

Note: Section 'C' contains 04 Long Answer Type Questions. Examinees will have to answer **any two** (02) questions. Each question is of 16 marks. Examinees have to delimit each answer in maximum 500 words.

10) (i) Solve $r = a^2 t$ by Monge's method.

(ii) Find the eigenvalues and eigenfunctions for the boundary value problem $y'' - 2y + \lambda y = 0;\ y(0) = 0,\ y(\pi) = 0$

11) State and prove Euler-Lagrange's equation.

11) (i) If m is a positive integer and $|x| > 1$, show that

$$ {}_2F_1\left(\frac{m+1}{2}, \frac{m+2}{2}; 1; -\frac{1}{x^2}\right) = \frac{(-1)^m x^{m+1}}{m!} \frac{d^m}{dx^m}\left(\frac{1}{\sqrt{x^2+1}}\right) $$

(ii) Show that

$$ \int_0^{\pi} x^2 P_{n+1} P_{n-1}\, dx = \frac{2n(n+1)}{(2n-1)(2n+1)(2n+3)} $$

13) (i) State and prove Rodrigues formula for Hermite polynomial

(ii) Prove that

$$ \frac{d}{dx}\left\{\frac{J_{-n}(x)}{J_n(x)}\right\} = -\frac{2 \sin n\pi}{\pi\, x\, J_n^2} $$