MA/MSCMT-02

June - Examination 2016

M.A./M.Sc.(Previous) Mathematics Examination

Real Analysis and Topology

Paper - MA/MSCMT-02

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C. Write answer as per the given instructions.

Section - A $8 \times 2 = 16$

(Very Short Answer Questions)

Note: Section 'A' contain 08 Very Short Answer Type Questions. Examinees have to attempt **all** questions. Each Question is of 02 marks and maximum word limit may be thirty words.

1) (i) Define σ Algebra.

(ii) Define outer measure of a set.

(iii) Define Borel measurable function.

(iv) State Fatou' lemma for measurable function.

(v) Define Hilbert space.

(vi) Define neighbourhood of a point x.

(vii) Define local base at any point of a Topological space.

(viii) Write finite intersection property (FIP) for compact spaces.

Section - B **$4 \times 8 = 32$**

(Short Answer Type Questions)

Note: Section 'B' contain 08 Short Answer Type Questions. Examinees will have to answer **any Four** (04) questions. Each question is of 08 marks. Examinees have to delimit each answer in maximum 200 words.

2) Prove that a non-empty collection R of subsets of a set X is an algebra of sets If and only If R is a ring of sets and $X \in R$.

3) Prove that the collection M of measurable sets is a σ - algebra.

4) Prove that If $< f_n >$ is a convergent sequence of measurable functions defined on measurable set E, then the limit function of $< f_n >$ is measurable.

5) State and prove Minkowski's inequality.

6) Let $D \subset L_2$ be everywhere dense in L_2. If Parseval's identity holds for all functions in D, then prove that the system $\{\phi_i\}$ is closed.

7) Prove that a second countable space is always first countable but converse is not true.

8) Prove that a topological space is a normal space iff for any closed set F and an open set G containing F, there exist an open set V such that $F \subset V \subset \overline{V} \subset G$.

9) Prove that continuous image of a connected space is connected.

Section - C **2 × 16 = 32**

(Long Answer Type Questions)

Note: Section 'C' contain 04 Long Answer Type Questions. Examinees will have to answer any Two (02) questions. Each question is of 16 marks. Examinees have to delimit each answer in maximum 500 words. Use of non-programmable scientific calculator is allowed in this paper.

10) (i) Prove that a topological space X is hausdroff space Iff every convergent filter on X has unique limit.

(ii) State and prove Alexander subbase lemma for compact topological space.

11) (i) Let R be the set of all real numbers show that the set $S = \{(x, y) ; x^2 + y^2 = 1\}$ in R^2 is the one point compactification of R and that $\infty = (0, 1)$ is the point at infinity.

(ii) Prove that every closed subspace of a locally compact space is locally compact.

12) (i) If $f(x)$ and $g(x)$ be two non-negative measurable functions of the set E, If $h(x) = f(x) + g(x)$ then prove that

$$\int_{E} h(x)\,dx = \int_{E} f(x)\,dx + \int_{E} g(x)\,dx$$

(ii) State and prove Cauchy-Bunyakowski-Schwarty (CBS) in equality for space L_2.

13) Prove that the necessary and sufficient condition for a bounded function f defined on interval $[a, b]$, to be L-integrable over $[a, b]$ is that for given $\in > 0$, there exists a measurable partition P of $[a, b]$ such that $U(f, p) - L(f, p) < \in$.