12. State & prove orthogonal property for Bessal function.

13. (i). Find the curve with fixed boundary revolves such that its rotation about x-axis generated minimal surface area.

(ii). Obtain the surface of minimum area, stretched over a given closed curve C, enclosing the domain D in the xy plane.

M.A./ M.Sc. Previous Examination, June 2015

MATHEMATICS

Paper: MT-03

(Differential equation, Calculus of Variation & Special Functions)

Time Allowed: 3 Hours

Maximum Marks: 80

Note:- The Question paper is divided into three sections A, B, and C. Use of calculator is allowed in this paper.

Section- A

Section 'A' contain 08 Very Short Answer Type Questions. Examinees have to attempt all questions. Each question is of 02 marks and maximum word limit is thirty words.

1. (i). Write two dimensional wave equation.

(ii). Write the Monge's subsidiary equations for PDE $pt - qs = q^3$

(iii). Write the Euler-Lagrange equation for the following functional

$$\int_0^1 \{y'^2 + 12xy\} dx$$

(iv). Define regular singular point.

(v). Write the value of $2F_1\left(\frac{1}{2}, 1; \frac{3}{2}; z^2\right)$, For $|z| < 1$.

(vi). Write Integral representation for confluent hyper geometric functions.

(vii). Write the value of $P_n(-1)$.

(viii). Write the value of $J'_0(x)$

Section- B

Section 'B' contain 08 Short Answer Type Questions. Examinees will have to answer any four (04) questions. Each question is of 08 marks. Examinees have to delimit each answer in maximum 200 words.

2. Find the general solution of the Riccati's equation.

$$\frac{dy}{dx} = 2 - 2y + y^2$$

3. Solve :

$$(e^x y + e^z)dx + (e^y z + e^x)dy + (e^y - e^x y - e^y z)dz = 0$$

4. Solve the following P.D.E.:

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial z}{\partial y}, \quad 0 < x < \pi, \ y > 0$$

Satisfy the boundary conditions :

(i) $z = 0 \ when \ x = 0$

(ii) $z = 0 \ when \ x = \pi$

(iii) $z = sin\, 3x \ when \ y = 0$

5. Find eigen values and eigen functions for the following boundary value problem.

$$y'' + 2y' + \lambda y = 0 \quad ; \ y(0) = 0, \ y(\pi) = 0$$

6. Test for an external of the functional:

$$F[y(x)] = \int_0^{\pi/2} (x^2 - y^2)dx, \ y(0) = 0, y\left(\frac{\pi}{2}\right) = 1$$

7. Solve in series :

$$x^2 \frac{d^2y}{dx^2} + (x + x^2)\frac{dy}{dx} + (x - 9)y = 0$$

8. If $|z| < 1 \ and \ \left|\frac{z}{1-z}\right| < 1 \ then$ prove that

$$2f_1(a, b \,; c \,; z) = (1 - z)^{-a} \ \ 2f_1(a, c - b, \quad ; c; \frac{z}{z-1})$$

9. Show that:

$$n\, P_n(x) = x\, P_n'(x) - P_{n-1}'(x)$$

Section- C

Section 'C' contain 04 Long Answer Type Questions. Examinees will have to answer any two (02) questions. Each question is of 16 marks. Examinees have to delimit each answer in maximum 500 words.

10. Show that :

(i). $\int_0^t x^{\frac{1}{2}} (1 - x)^{-\frac{1}{2}} [1 - x^2(t - x)^2]^{-\frac{1}{2}} dx = \frac{1}{2}\pi t \ 2f_1\left[\frac{1}{4}, \frac{3}{4}; 1 \,; \frac{t^4}{16}\right]$

(ii). $\int_{-1}^{1} x^2 P_{n+1} P_{n-1}\, dx = \frac{2n\,(n+1)}{(2n-1)(2n+1)(2n+3)}$

11. Show that :

(i) $nQ_{n+1}' + (n + 1)Q_{n-1}' = (2n + 1)x\, Q'_n$

(ii) $(2n + 1)x\, Q_n = (n + 1)\, Q_{n+1} + nQ_{n-1}$