M.A./ M.Sc. Previous Examination, June 2015

MATHEMATICS

Paper: MT-02

(Real Analysis and Topology)

Time Allowed: 3 Hours
Maximum Marks: 80

Note:- The Question paper is divided into three sections A, B, and C. Use of calculator is allowed in this paper.

Section- A

Section 'A' contain 08 Very Short Answer Type Questions. Examinees have to attempt all questions. Each question is of 02 marks and maximum word limit is thirty words.

1. (i). State Minkowski's inequality.
   (ii). What is comparable topologies?
   (iii). Define base for a topology.
   (iv). Define T_1- space.
   (v). Define locally connected space at a point.
   (vi). What is Embedding?
   (vii). Define a σ-ring.
   (viii). Define eventually net.

Section- B

Section 'B' contain 08 Short Answer Type Questions. Examinees will have to answer any four (04) questions. Each question is of 08 marks. Examinees have to delimit each answer in maximum 200 words.

2. Let $X = \{1, 2, 3, 4\} and\ C = \{\{1, 2\}, \{1, 3\}\}$, then find base and filter taking C as a sub-base.
3. Let F be a finitely short family of open sets of a topological space (X, τ). Then prove that ∃ a maximal finitely short sub family M of τ such that $F \subset M$.
4. Prove that every open continuous image of a locally compact space is locally compact.
5. Show that every T_3-space is a T_2-space.
6. Show that a function $f : X \rightarrow Y$ is continuous iff the inverse image of every closed set of Y is a closed subset of X.
7. Let $< f_n >$ be a sequence of functions belonging to L^p-space. Then prove that if this sequence is convergent, then it is a Cauchy sequence.
8. Show that the lower Lebesgue Darboux sums of any bounded measurable function f on a measurable set E cannot exceed its upper Lebesgue Darboux sums.
9. Let $< E_i >$ be an infinite increasing sequence of measurable sets, then prove that :

$$m^*\left(\bigcup_{i=1}^{\infty} E_i\right) = \lim_{n\to\infty} m(E_n)$$

Section- C

Section 'C' contain 04 Long Answer Type Questions. Examinees will have to answer any two (02) questions. Each question is of 16 marks. Examinees have to delimit each answer in maximum 500 words.

10. (i). Prove that Every p[th] power of summable function on set E is summable on E i.e. $L^p[E] \subset [E]$ but the converse is not true.

    (ii). Show that second countable space is always first countable but converse is not true.
11. (i). Prove that a topological space (X, τ) *is a* normal space iff for any closed set F and an open set G containing F, there exist on open set V s.t. $F \subset V \subset \bar{V} \subset G$.

    (ii). Prove that every second countable regular space is normal space.
12. (i). Show that τ_{∞} is a topology on X_{∞}.

    (ii). Show that a subset of R is connected iff it is an interval.
13. (i). Prove that a topological space X is hausdorff space iff every convergent filter on X has a unique unit.

    (ii). Prove that union of arbitrary family of connected subset of a topological space is connected if family has non-empty intersection.