M.A./ M.Sc. Previous Examination,

June 2015

MATHEMATICS

Paper: MT-01(Advanced Algebra)

Time Allowed: 3 Hours
Maximum Marks: 80

Note:- The Question paper is divided into three sections A, B, and C. Use of calculator is allowed in this paper.

Section- A

Section 'A' contain 08 Very Short Answer Type Questions. Examinees have to attempt all questions. Each question is of 02 marks and maximum word limit is thirty words.

1. (i). Define internal direct product.
   (ii). Define commutator of two elements of a group.
   (iii). Define prime element in an integral domain.
   (iv). Define Galois group.
   (v). Define eigenvectors of a linear transformation.
   (vi). Write Schwartz inequality.
   (vii). Define characteristics polynomial of a square matrix.
   (viii). Define normal extension of a field.

Section- B

Section 'B' contain 08 Short Answer Type Questions. Examinees will have to answer any four (04) questions. Each question is of 08 marks. Examinees have to delimit each answer in maximum 200 words.

2. If $f: M \rightarrow M^*$ as an R-module Homomorphism. Then prove that f is a monomorphism if and only if $Ker\ (f) = \{0\}$.
3. Let V be an n-dimensional vector space over a field F. Prove that V is isomorphic to the vector space F^n.
4. Let K be a Galois extension of a field F and Characteristic of F be zero. Then prove that the fixed field under the Galois group G(K/F) is F itself.
5. State and prove Cramer's rule.
6. If $S = \{v_1, v_2, \dots \dots v_n\}$ is an orthonormal set in an Inner product space V. Then prove that for any vector $v \in V$, vector $u = v - \sum_{i=1}^{n} v_1 < v, v_i >$ is orthogonal to each of the vector $v_1, v_2, \dots \dots v_n$ and consequently to the subspace spanned by S.
7. If both t and s are self adjoint linear transformation of an inner product space V. Then prove that $ts + st$ is self adjoint. If both t and s are skew adjoint then prove that $ts - st$ is skew adjoint.
8. Let V and V' be inner product spaces. Then prove that a linear transformation $t : V \rightarrow V'$ is orthogonal is and only if

$$\|t(u)\| = \|u\| \quad \forall u \in V$$

9. Let R be a Euclidean ring, then prove that every non zero element in R is either a unit or can be written as product of a finite number of prime elements of R.

Section- C

Section 'C' contain 04 Long Answer Type Questions. Examinees will have to answer any two (02) questions. Each question is of 16 marks. Examinees have to delimit each answer in maximum 500 words.

10. Let V and V' be two vector spaces over the same field F and $B = \{b_1, b_2, \dots \dots b_n\}$ be a basis for V and $B' = \{b_1', b_2', \dots \dots b_n'\}$ be a set of vectors in V' if $t : V \rightarrow V'$ be a linear transformation such that $t(b_i) = b_i'$, $i = 1, 2, \dots \dots n$. Then prove that t is an isomorphism iff the set B' is a basis for V'.
11. (i). Show that the map $t : V_2R \rightarrow V_3R$ defined by $t(a, b) = (a + b, a - b, b)$ is a linear transformation. Find range, rank, null space and nullity of t.
(ii). Prove that every additive abelian group is a module over the ring Z of integers.
12. (i). Let $t : R^3 \rightarrow R^3$ be a linear transformation such that $t(a, b, c) = (3a + c, -2a + b, -a + 2b + 4c)$. Write the matrix of t in the ordered basis $\{\alpha_1, \alpha_2, \alpha_3\}$ where $\alpha_1 = (1,0,1), \alpha_2 = (-1,2,1), \alpha_3 = (2,1,1)$
(ii). Prove that the necessary and sufficient condition for a non-void subset N of an R-module M over a ring R with unity to be a submodule of M is that rx+sy $\in N \quad \forall r, s \in R \quad \forall x, y \in N$.
13. (i). Prove that every finite extension of a field is an algebraic extension but the converse is not necessarily true.
(ii). Let K be an extension of a field F. Then prove that the elements in K which are algebraic over f form a subfield of K.