12. (i). Prove that the characteristic numbers of a symmetric kernel are real.

(ii). Solve the integral equation:

$$g(x) = e^{-x} - 2\int_0^x \cos(x-t)g(t)dt$$

13. (i). State and prove Hilbert-Schmidt theorem.

(ii). Using Hilbert-Schmidt method, solve integral equation

$$g(x) = 1 + \lambda \int_0^1 k(x,t)\, g(t)dt$$

Where $k(x,t) = \begin{cases} x\,(t-1);\ 0 \le x \le t \\ t(x-1),\ t \le x \le 1 \end{cases}$

M.Sc. Final Examination, June 2015

MATHEMATICS

Paper: MSCMT-09

(Integral Transforms and Integral Equations)

Time Allowed: 3 Hours
Maximum Marks: 80

Note:- The Question paper is divided into three sections A, B, and C. Use of calculator is allowed in this paper.

Section- A

Section 'A' contain 08 Very Short Answer Type Questions. Examinees have to attempt all questions. Each question is of 02 marks and maximum word limit is thirty words.

1. (i). Define Error function and complementary Error function.

(ii). Write the change of scale property for Inverse laplace transform.

(iii). State the mellin inversion theorem.

(iv). What is the difference between Linear and Non-linear Integral equation.

(v). Define symmetric kernel of an integral equation.

(vi). Define the Abel integral equation.

(vii). Define orthogonal system of functions.

(viii). State Hilbert-Schmidt theorem.

Section- B

Section 'B' contain 08 Short Answer Type Questions. Examinees will have to answer any four (04) questions. Each question is of 08 marks. Examinees have to delimit each answer in maximum 200 words.

2. Find Laplace Transform of $t^2.u(t-3)$, where $u(t-3)$ is a unit step function.
3. Find fourier cosine transform of $f(x) = \frac{1}{1+x^2}$.
4. Prove that:

$$M\{(1+x^a)^{-b}\,;p\} = \frac{\Gamma\left(\frac{p}{a}\right)\,\Gamma\left(b-\frac{p}{a}\right)}{a\,\Gamma(b)}\,;\ 0 < Re\,(p) < Re\,(ab).$$

5. Find the Hankel transform of the function :

$$f(x) = \begin{cases} x^n, & 0 < x < a \\ 0, & x > a \end{cases} \qquad (n > -1)$$

taking $x\,J_n\,(px)$ as the kernel.

6. Solve the homogeneous fredholm integral equation of the second kind.

$$g(x) = \lambda \int_0^{2\pi} \sin(x+t)\ g(t)dt$$

7. Using the method of successive approximation solve the integral equation :

$$g(x) = 1 + \int_0^x (x-t)g(t)dt \ \ taking\ g_0(x) = 0$$

8. Show that if the sequence $\{g_k(x)\}$ be all the eigen functions of a symmetric L_2 –kernel with $\{\lambda_k\}$ as the corresponding eigen values. Then the series : $\sum_{n=1}^{\infty} \frac{|g_n(x)|^2}{\lambda^2 n}$ converges and its sum is bounded by C_1^2 which is an upper bound of the integral:

$$\int_a^b |k^2(x,t)|\,dt$$

9. By means of resolvent kernel find the solution of :

$$g(x) = e^x \sin x + \int_0^x \frac{2+\cos x}{2+\cos t}\,g(t)dt$$

Section- C

Section 'C' contain 04 Long Answer Type Questions. Examinees will have to answer any two (02) questions. Each question is of 16 marks. Examinees have to delimit each answer in maximum 500 words.

10. (i). Show that

$$L^{-1}\left[\frac{p^2}{p^4+4a^4}\right] = \frac{1}{2a}[\cos hat \sin at + \sin hat \cos at]$$

(ii) Find the fourier transform of $f(t)$, where $f(t) = \begin{cases} 1-t^2, & |t| < 1 \\ 0, & |t| > 1 \end{cases}$ and hence evaluate $\int_0^\infty \left(\frac{t\cos t - \sin t}{t^3}\right)\cos\frac{t}{2}\,dt.$

11. Prove that :

$$H_v\{e^{-px^2/4} f(x); s\} = 2L\{f(2\sqrt{x}\ J_v(2s\sqrt{x}); p\}$$

Deduce that $H_v\left\{x^v e^{-p\frac{x^2}{4}}; s\right\} = \frac{2^{v+1}s^v}{p^{v+1}} e^{-\frac{s^2}{p}}$

and hence that:

(i) $H_v\left\{x^v e^{-\frac{x^2}{a^2}}; s\right\} = \left(\frac{a^2}{2}\right)^{v+1} e^{-a^2\frac{s^2}{4}}$

(ii) $H_v\left\{x^v e^{-\frac{x^2}{2}}; s\right\} = s^v e^{-\frac{s^2}{2}}$