(ii). Find the angle between two tangential direction on the surface in the terms of direction ratio.

M.A./ M.Sc. Previous Examination, June 2015

MATHEMATICS

Paper: MT-04

(Differential Geometry & Tensors)

Time Allowed: 3 Hours
Maximum Marks: 80

Note:- The Question paper is divided into three sections A, B, and C. Use of calculator is allowed in this paper.

Section- A

Section 'A' contain 08 Very Short Answer Type Questions. Examinees have to attempt all questions. Each question is of 02 marks and maximum word limit is thirty words.

1. (i). Define right helicoid.
(ii). Define Normal section and oblique section of a surface.
(iii). Define the lines of curvature.
(iv). Define Geodesic.
(v). State Clairut's theorem.
(vi). Write the Weingarten formulae.
(vii). State Gauss-Bonnet theorem.
(viii). Write Gauss's characteristic equations.

Section- B

Section 'B' contain 08 Short Answer Type Questions. Examinees will have to answer any

four (04) questions. Each question is of 08 marks. Examinees have to delimit each answer in maximum 200 words.

2. Find the equation of the developable surface which contains the two curves $y^2 = 4ax, z = 0$ and $(y-b)^2 = 4cz, x = 0$ and also show that its edge regression lies on the surface:

$$(ax + by + cz)^2 = 3abx\,(b + y)$$

3. State and prove Meunienis theorem.
4. Find the Geodesic curvature of the curve u = constant on the surface $x = u\cos\theta\,, y = u\sin\theta\,, z = \frac{1}{2}au^2$
5. From the Gauss characteristic equation deduce that, when the parametric curves are orthogonal :

$$k = \frac{1}{\sqrt{EG}}\left[\frac{\partial}{\partial u}\left(\frac{1}{E}\frac{\sqrt{G}}{\partial u}\right) + \frac{\partial}{\partial v}\left(\frac{1}{\sqrt{G}}\frac{\partial\sqrt{E}}{\partial v}\right)\right]$$

6. State and prove Bonner's theorem on parallel surfaces.
7. Prove that :

$$A^{i,j}_{j} = \frac{1}{\sqrt{g}}\frac{\partial}{\partial x^j}\left(\sqrt{g}\,A^{ij}\right) + A^{ik}\begin{Bmatrix} i \\ jk \end{Bmatrix}$$

8. Prove that :

(a) $R^{\alpha}_{.ijk}$ has cyclic property in its subscripts i.e.

$R^{\alpha}_{.ijk} + R^{\alpha}_{.jkp} + R^{\alpha}_{.kij} = 0$

(b) $R^{\alpha}_{.ijk}$ vanish an contradiction in α and i i.e.

$R^{\alpha}_{.ijk} = 0$

9. Find the oscillating plane at the point 't' on the helix $x = a\cos t, y = a\sin t, z = ct$.

Section- C

Section 'C' contain 04 Long Answer Type Questions. Examinees will have to answer any two (02) questions. Each question is of 16 marks. Examinees have to delimit each answer in maximum 500 words.

10. Prove that the necessary and sufficient condition for a space V_N to be flat is that the Riemann-Christoffel tensor be Identically Zero i.e. $R^{\alpha}_{.ijk} = 0$
11. Evaluate $div\,A^j$ in (i) cylindrical polar coordination (ii) Spherical polar coordinates.
12. (i). Show that the metric of a Euclidian space referred to cylindrical coordinates is given by :

$$ds^2 = (dr)^2 + (rd\theta)^2 + (dz)^2$$

Determine its metric tensor and conjugate metric tensor.

(ii). For a surface given by $ds^2 = \phi\,(du^2 + dv^2)$ prove that:

$$l = \frac{\phi_1}{2\phi}, m = \frac{\phi_2}{2\phi}, n = \frac{\phi_1}{2\phi}, \lambda = \frac{\phi_2}{2\phi}, \mu = \frac{\phi_1}{2\phi}, v = \frac{\phi_2}{2\phi}$$

And further show that mainradii-codazzi relation become:

$$L_2 - M_1 = \frac{1}{2}\frac{\phi_2}{\phi}\,(L + N), N_1 - M_2 = \frac{1}{2}\frac{\phi_1}{\phi}\,(L + N)$$

13. (i). Find the principal section and principal curvature of the surface $x = a\,(u + v), y = b\,(u - v), z = uv$.