10. Let G_1 and G_2 be groups. If N_1 and N_2 are normal subgroups of G_1 and G_2 respectively, then show that $N_1 \times N_2$ is normal in $G_1 \times G_2$ and

$$(G_1 \times G_2)/(N_1 \times N_2) \cong (G_1/N_1) \times (G_2/N_2)$$

11. Show that every Euclidean domain is a unique factorization domain.

12. Show that there exists a multi-linear map det : $(F^n)^n \to F$ such that :

$$\det\,(A_1, A_2, \ldots\ldots\ldots, A_n) = \sum_{\sigma \in s_n} \in(\sigma)\, a_{\sigma(1)_1} a_{\sigma(2)_2} \ldots . a_{\sigma(n)_n}$$

for all $A_i \in F^n$ and $A = (A_1, A_2 \ldots\ldots A_n) = [a_{ij}]$ satisfying the axioms of determinant function.

13. State and prove principal axis theorem.

MAMT-01/MSCMT-01

June – Examination 2024

M.A./M.Sc. (Previous) Examination

MATHEMATICS

(Advanced Algebra)

Paper : MAMT-01/MSCMT-01

Time : 3 Hours] [*Maximum Marks :* ***80***

Note :– The question paper is divided into three Sections A, B and C. Write answers as per the given instructions. Use of non-programmable Scientific Calculator is allowed in this paper.

Section–A **8×2=16**

(Very Short Answer Type Questions)

Note :– Answer all questions. As per the nature of the question, delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries 2 marks.

1. (i) Define internal direct product.

(ii) Define derived subgroup.

(iii) Define quotient module.

(iv) Define dual basis.

(v) Define normal field extention.

(vi) Define Eigen Vector.

(vii) Define orthogonal set.

(viii) Define adjoint of a linear map.

Section–B **4×8=32**

(Short Answer Type Questions)

Note :– Answer any *four* questions. Each answer should not exceed **200** words. Each question carries 8 marks.

2. Let G be a group and N(a) be normalizer of $a \in$ G. Then show that two elements $x, y \in$ G give rise to the same conjugate of a if and only if they belong to the same right coset of N(a) in G.

3. If G is a solvable group, then show that every subgroup of G is also solvable.

4. Let R be a Euclidean ring and a, b be any non-zero elements in R. If b is not a unit in R, then show that $d(a) < d(ab)$.

5. If $t : v \to v'$ is an isomorphism then show that $\{v_1, v_2, \ldots\ldots, v_n\}$ is linearly independent if and only if $\{t(v_1), t(v_2) \ldots\ldots, t(v_n)\}$ is linearly independent.

6. If F is a field, then every polynomial $f(x) \in F[x]$ has a splitting field.

7. Show that an $n \times n$ matrix A over a field F is invertible if and only if rank (A) = n.

8. Let V be an inner product space and $u, v \in$ V be orthogonal to each other, then show that :

$$\|u+v\|^2 = \|u\|^2 + \|v\|^2$$

9. Let V be a finite dimensional inner product space. Then show that a linear transformation $t : V \to V$ is orthogonal if and only if its matrix relative to an orthonormal basis is orthogonal.

Section–C **2×16=32**

(Long Answer Type Questions)

Note :– Answer any *two* questions. You have to delimit your each answer maximum up to **500** words. Each question carries 16 marks.