# MAMT–10/MSCMT–10

## June – Examination 2024

## M.A./M.Sc. (Final) Examination

### MATHEMATICS

### (Mathematical Programming)

### Paper : MAMT–10/MSCMT–10

*Time : **3** Hours* ] [ *Maximum Marks : **80***

***Note*** :– The question paper is divided into three Sections A, B and C. Write answers as per the given instructions.

**Section–A** **8×2=16**

**(Very Short Answer Type Questions)**

***Note*** :– Answer all questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries 2 marks.



1. (i) Show that :

$$f(x) = 2x_1^2 + x_2^2$$

is a convex function over $R^2$.

(ii) Define bounded variable linear programming problem.

(iii) Write the condition when a point will be Saddle point in Lagrangian function.

(iv) Define unconstrained optimization problem.

(v) Write the following quadratic form in matrix vector notation :

$$\begin{pmatrix} 1 & 2 & 4 \\ 2 & 6 & -2 \\ 4 & -1 & 14 \end{pmatrix}$$

(vi) State Bellmen's principle of optimality.

(vii) Define feasible point for the Dual.

(viii) Write Dual of Max :

$$f(X) = C^T X + \frac{1}{2} X^T GX$$

subject to $AX = b$; $C \geq 0$.

## Section–B 4×8=32

**(Short Answer Type Questions)**

***Note*** :– Answer any *four* questions. Each answer should not exceed **200** words. Each question carries 8 marks.

2. Prove that a hyperplane is a closed set.

3. Solve the following L.P.P. with the help of revised simplex method but without use of artificial variables :

   Max. :

$$Z = 2x_1 - 6x_2$$

   Subject to :

$$x_1 - 3x_2 \leq 6$$

$$2x_1 + 4x_2 \geq 8$$

$$-x_1 + 3x_2 \leq 6$$

$$x_1, x_2 \geq 0$$



4. Use branch and bound method to solve the following L.P.P. :

   Maximize : $Z = 7x_1 + 9x_2$

   Subject to :

$$-x_1 + 3x_2 \leq 6$$

$$7x_1 + x_2 \leq 35$$

$$x_2 \geq 7$$

5. Obtain the necessary and sufficient conditions for the optimum solution of the following N.L.P.P. :

   Minimize : $Z = 4x_1^2 + 2x_2^2 + x_3^2 - 4x_1x_2$

   Subject to :

$$x_1 + x_2 + x_3 = 15$$

$$2x_1 - x_2 + 2x_3 = 20$$

$$x_1, x_2, 2x_3 \geq 0$$

6. Solve the following non-linear programming problem graphically :

Max. :

$$f(x_1, x_2) = 8x_1 + 8x_2 - x_1^2 - x_1^2$$

Subject to :

$$x_1 + x_2 \leq 12$$

$$x_1 - x_2 \geq 4$$

$$x_1, x_2 \geq 0$$

7. Test the definiteness of the quadratic form :

$$X^T AX = (x_1, x_2, x_3) \begin{bmatrix} 3 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

8. Solve the following quadratic programming problem by Wolfe's method :

Min. :

$$f(x_1, x_2) = -10x_1 - 25x_2 + 10x_1^2 + x_2^2 + 4x_1x_2$$



Subject to :

$$x_1 + 2x_2 \leq 10$$

$$x_1 + x_2 \leq 9$$

$$x_1, x_2 \geq 0$$

9. Use dynamic programming to solve the following L.P.P. :

Max. :

$$Z = 2x_1 + 5x_2$$

Such that :

$$2x_1 + x_2 \leq 43$$

$$2x_2 \leq 46$$

and $$x_1, x_2 \geq 0$$

**Section–C** **2×16=32**

**(Long Answer Type Questions)**

***Note*** :– Answer any *two* questions. You have to delimit your each answer maximum up to **500** words. Each question carries 16 marks.

10. Solve the following linear programming problem by revised simplex method :

Max. :

$$Z = 2x_1 + x_2$$

Subject to :

$$3x_1 + 4x_2 \leq 6$$

$$6x_1 + x_2 \leq 3$$

$$x_1, x_2 \geq 0$$

11. Use Beale's method to solve the following quadratic programming problem :

Minimize :

$$f(x_1, x_2) = 6 - 6x_1 + 2x_1^2 - 2x_1x_2 + 2x_2^2$$

Subject to :

$$x_1 + x_2 \leq 2$$

$$x_1, x_2 \geq 0$$



12. Find the optimum integer solution to the following L.P.P. :

Max. : $Z = 3x_1 + 4x_2$

Subject to :

$$3x_1 + 2x_2 \leq 8$$

$$x_1 + 4x_2 \leq 10$$

$x_1, x_2 \geq 0$, and are integers.

13. Solve the following convex separable programming problem :

Min. : $Z = x_1^2 - 2x_1 - x_2$

Such that :

$$2x_1^2 + 3x_2^2 \leq 6$$

and $x_1, x_2 \geq 0$