10. Solve :

$$\frac{d^4y}{dx^4} - y = 1$$

Subject to the conditions $y(0) = y'(0) = y''(0) = y'''(0) = 0$.

11. Solve the following partial differential equation with Fourier transform :

$$\frac{\partial V}{\partial t} = \frac{\partial^2 V}{\partial x^2}, x > 0, t > 0$$

Subject to conditions :

(i) $V = 0$ when $x = 0$, $t > 0$

(ii) $V = f(x) = \begin{cases} 1, & 0 < x < 1 \text{ when } t = 0 \\ 0, & x \geq 1 \end{cases}$

$V(x, t)$ is bounded $x > 0$, $t > 0$.

12. Solve by method of successive approximations :

$$g(x) = \left(\frac{3}{2}e^x - \frac{1}{2}xe^x - \frac{1}{2}\right) + \frac{1}{2}\int_0^1 tg(t)dt$$

13. Find the Eigne values and Eigen functions of the homogeneous integral equation :

$$g(x) = \lambda \int_0^1 K(x, t)g(t)dt,$$

where :

$$K(x,t) = \begin{cases} x(t-1), 0 \leq x \leq t \\ t(x-1), t \leq x \leq 1 \end{cases}$$

MAMT–09/MSCMT–09

June – Examination 2024

M.A./M.Sc. (Final) Examination

MATHEMATICS

(Integral Transforms and Integral Equations)

Paper : MAMT–09/MSCMT–09

*Time : **3** Hours*] [*Maximum Marks : **80***

Note :– The question paper is divided into three Sections A, B and C. Write answers as per the given instructions. Use of non-programmable scientific calculator is allowed in this paper.

Section–A **8×2=16**

(Very Short Answer Type Questions)

Note :– Answer all questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries 2 marks.

1. (i) Write Dirichlet's conditions.

(ii) Write the Laplace transform of the function $f(t) = e^{4t} \cos 4t$.

(iii) Define Fourier sine transform.

(iv) Write relationship between Fourier transform and Laplace transform.

(v) Define Fredholm integral equation of second kind.

(vi) Define resolvent kernel.

(vii) Define Norm of a complex function.

(viii) What is Neumann series ?

Section–B **4×8=32**

(Short Answer Type Questions)

Note :– Answer any *four* questions. Each answer should not exceed **200** words. Each question carries 8 marks.

2. Find the Inverse Laplace transform of :

$$\frac{4}{s-3}-\frac{4}{s^2+4}+\frac{s-4}{(s-4)^2-4}$$

3. Solve $(D^2 + 9)\, y = \cos 2t$.

Given that :

$$y(0)=1, y\left(\frac{\pi}{2}\right)=-1$$

4. Find the Fourier cosine transform of e^{-t^2}.

5. State and prove convolution theorem for Mellin Transform.

6. Show that the function $g(x) = xe^x$ is a solution of the Volterra integral equation :

$$g(x)=\sin x+2\int_0^x \cos(x-t)\, g(t)dt$$

7. Find the resolvent kernels of the following kernel :

$K(x, t) = (1 + x)(1 - t)$; $a = -1$, $b = 0$

8. Prove that, if a kernel is symmetric, then all its iterated kernels are also symmetric. For the integral equation :

$$g(x)=f(x)+\lambda\int_a^b K(x,t)g(t)dt,$$

find $D(\lambda)$ and $D(x, t ; \lambda)$ for the kernel :

$K(x, t) = \sin x$; $a = 0$, $b = \pi$.

9. Find the resolvent kernel and solution of :

$$g(x)=f(x)+\lambda\int_0^1 (x+t)g(t)dt$$

Section–C **2×16=32**

(Long Answer Type Questions)

Note :– Answer any *two* questions. You have to delimit your each answer maximum up to **500** words. Each question carries 16 marks.