Section–C 2×16=32

(Long Answer Type Questions)

Note :– Answer any *two* questions. You have to delimit your each answer maximum up to **500** words. Each question carries 16 marks.

10. State and prove Risez representation theorem in Hilbert space.

11. If B and B′ be Banach spaces and T is a continuous linear transformation of B onto B′, then prove that the image of every open sphere centred at origin in B contains an open sphere centred at origin in B′.

12. State and prove spectral theorem for finite dimensional Hilbert space.

13. State prove inverse function theorem.

MAMT–06/MSCMT–06

June – Examination 2024

M.A./M.Sc. (Final) Examination

MATHEMATICS

(Analysis and Advanced Calculus)

Paper : MAMT–06/MSCMT–06

*Time : **3** Hours*] [*Maximum Marks : **80***

Note :– The question paper is divided into three Sections A, B and C. Write answers as per the given instructions.

Section–A 8×2=16

(Very Short Answer Type Questions)

Note :– Answer all questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries 2 marks.

1. (i) Explain convergence in normed linear space.

   (ii) What do you mean by Conjugate space ?

   (iii) State Parseval's identity for a Hilbert space.

   (iv) Write Taylor's formula with Lagrange's remainder.

   (v) Define Self Adjoint Operator.

   (vi) Define Inner Product space.

   (vii) Describe the Quotient space.

   (viii) Define integral solution of the differential equation.

Section–B **4×8=32**

(Short Answer Type Questions)

Note :– Answer any *four* questions. Each answer should not exceed **200** words. Each question carries 8 marks.

2. State and prove Minkowaski's inequality for C^n.

3. If x, y are any *two* vectors in a Hilbert space H, then prove that :

$$4(x,y)=\|x+y\|^2-\|x-y\|^2+i\|x+iy\|^2-i\|x-iy\|^2$$

4. Prove that every Hilbert space is reflexive.

5. If P is a projection on a Hilbert space H with range M and null space N then prove that $M \perp N$ if and only if P is selp adjoint and in this case :

$$N = M^{\perp}$$

6. State and prove Schwarz inequality for an inner product space.

7. State and prove Global uniqueness theorem.

8. Let f be a regulated function on a compact interval $[a, b]$ or R into R such that $a < b$ and for all t in $[a, b]$, $f(t) \geq 0$. Then prove that :

$$\int_a^b f(t)\,dt \geq 0$$

   Further prove that if f be a continuous function at a point c of $[a, b]$ and $f(c) > 0$, then :

$$\int_a^b f(t)\,dt > 0$$

9. Show that every compact subset of a normed linear space is bounded but its converse need not be true.