MAMT–05/MSCMT–05

June – Examination 2024

M.A./M.Sc. (Previous) Examination

MATHEMATICS

(Mechanics)

Paper : MAMT–05/MSCMT-05

*Time : **3** Hours*] [*Maximum Marks : **80***

Note :– The question paper is divided into three Sections A, B and C. Write answers as per the given instructions. Use of non-programmable scientific calculator is allowed in this paper.

Section–A **8×2=16**

(Very Short Answer Type Questions)

Note :– Answer all questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries 2 marks.

1. (i) State theorem of parallel axes for moment of inertia.

   (ii) What do you mean by an axis of spontaneous rotation ?

   (iii) Write down the vector form of Euler's equations of motion.

   (iv) State principle of conservation of linear momentum.

   (v) State the Bernoulli's theorem.

   (vi) Define compressibility of a fluid.

   (vii) Write equation of continuity in vector form.

   (viii) State Bernoulli's theorem for steady fluid motion.

Section–B **4×8=32**

(Short Answer Type Questions)

Note :– Answer any *four* questions. Each answer should not exceed **200** words. Each question carries 8 marks.

2. State and prove D'Alembert's Principle.

3. A cylinder rolls down a smooth plane whose inclination to the horizon is α, unwrapping as it goes a fine string fixed to the highest point of the plane, find its acceleration and the tension of the string.

4. Two link rods AB and BC, each of length $2a$ are freely joined at B, AB can turn round the rod A and C can move freely on a vertical straight line through A. Initially the rods are held in a horizontal line, C being in coincidence with A and they are then released. Show that when the rods are inclined at an angle θ to the horizontal, the angular velocity of either is :

$$\sqrt{\left\{\frac{3g}{a}.\frac{\sin\theta}{1+3\cos^2\theta}\right\}}$$

5. A top is executing steady motion with angular velocity n about its axis which is vertical, show that the motion is stable.

6. Show that for a body of revolution the maximum value of the angle between the axis of the impulsive couple acting on it and the instantaneous axis of initial motion set up by the couple in the body is :

$$\sin^{-1}\left(\frac{C-A}{C+A}\right)$$

7. A circular disc, of radius a, has a thin rod pushed through its centre perpendicular to its plane, the length of the rod being equal to the radius of the disc. Show that the system can not spin with the rod vertical unless the angular velocity is greater than $\sqrt{\frac{20g}{a}}$.

8. Derive Euler's dynamical equations of motion in Cartesian coordinates.

9. A mass of fluid moves in such a way that each particle describes a circle in one plane about a fixed axis. Show that the equation of continuity is :

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho w)}{\partial \theta} = 0$$

Where w be the angular velocity of a particle whose azimuthal angle is θ at time t.

Section–C **2×16=32**

(Long Answer Type Questions)

Note :– Answer any *two* questions. You have to delimit your each answer maximum up to **500** words. Each question carries 16 marks.

10. Two equal uniform rods AB and AC are freely hinged at A and rest in a straight line on a smooth table. A blow is struck at it perpendicular to the rods; show that the kinetic energy generated is $\frac{7}{4}$ times, what it would be if the rods were rigidly fastened together at A ?

11. A uniform vertical circular plate, of radius a; is capable of revolving about a smooth horizontal axis through its centre; a rough perfectly flexible chain, whose mass is equal to that of the plate and whose length is equal to its circumference, hangs over is rim in equilibrium, if one end be slightly displaced, show that the velocity of chain, when the end reaches the plate is $\sqrt{\frac{\pi a g}{6}}$.

12. State and prove Bernoulli's theorem.

13. A mass of liquid surrounds a solid sphere of radius a and its outer surface, which is a concentric spheres of radius b, is subject to a given constant pressure P, no other force being in action on the liquid. The solid sphere suddenly shrinks into a concentric sphere. Determine the subsequent motion and the impulsive action on the sphere.