Section–C   2×16=32

(Long Answer Type Questions)

Note :– Answer any *two* questions. You have to delimit your each answer maximum up to **500** words. Each question carries 16 marks.

10. Find the radii of curvature and torsion at a point of the curve :

$$x^2 + y^2 = a^2,\ x^2 - y^2 = az$$

11. Find the principal sections and principal curvatures of the surface :

$$x = a(u + v),\ y = b(u - v),\ z = uv$$

12. State and prove Meunier's theorem.

13. Show that the metric of a Euclideans space, referred to cylindrical coordinates is given by :

$$ds^2 = (dr)^2 + (r\,d\theta)^2 + (dz)^2$$

Determine its metric tensor and conjugate metric tensor.

MAMT–04/MSCMT–04

June – Examination 2024

M.A./M.Sc. (Previous) Examination

MATHEMATICS

(Differential Geometry and Tensor)

Paper : MAMT–04/MSCMT–04

*Time : **3** Hours*]   [*Maximum Marks : **80***

Note :– The question paper is divided into three Sections A, B and C. Write answers as per the given instructions. Use of non-programmable scientific calculator is allowed in this paper.

Section–A   8×2=16

(Very Short Answer Type Questions)

Note :– Answer all questions. As per the nature of the question delimit your answer in maximum up to **30** words. Each question carries 2 marks.

1. (i) Define curvature.

(ii) Define Osculating sphere.

(iii) Define principal section and principal direction.

(iv) Define metric of a surface.

(v) Write down the equation of the evolute.

(vi) Examine whether the parametric curves $x = b \sin u \cos v$, $y = b \sin u \sin v$, $z = b \cos u$ on a sphere of radius b constitute an orthogonal system.

(vii) State Gauss-Bonnet theorem.

(viii) Define conjugate metric tensor.

Section–B **4×8=32**

(Short Answer Type Questions)

Note :– Answer any *four* questions. Each answer should not exceed **200** words. Each question carries 8 marks.

2. Prove that if the circle $lx + my + nz = 0$, $x^2 + y^2 + z^2 = 2cz$ has three point contact at the origin with the paraboloid $ax^2 + by^2 = 2z$, then $c = (l^2 + m^2n)(bl^2 + am^2)$.

3. Prove that the principal normals at consecutive points of a curve do not interest unless $\tau = 0$, where τ is torsion.

4. Find the equation to the edge of regression of the developable surface :

$$y = xt = t^3,\ z = t^3 y - t^6$$

5. On the paraboloid $x^2 - y^2 = z$, find the orthogonal trajectories of the sections by the planes z = constant.

6. Find the asymptotic lines on the surface $z = y \sin x$.

7. Prove that the law of transformation of a contravariant vector is transitive.

8. Evaluate div A^j in cylindrical polar coordinates.

9. Show that the divergence of Einstein tensor vanishes.