Section–C **2×16=32**

(Long Answer Type Questions)

Note :– Answer any *two* questions. You have to delimit your each answer maximum up to **500** words. Each question carries 16 marks.

10. Write and establish necessary and sufficient condition for Integrability of the total differential equation.

11. (i) Solve $t - r\sec^4 y = 2q\tan y$ using Monge's method.

    (ii) Find the extremal of the functional $I = \int_0^1 (1 + y''^2)\,dx$, while :

    $$y_0 = 0, y'_0 = 1, y_1 = 1, y'_1 = 1 .$$

12. Solve the two dimensional heat conduction equation :

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{k}\frac{\partial u}{\partial t}$$

13. (i) Solve the Legendre's equation :

$$(1 - x^2)\frac{d^2 y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0$$

    (ii) Prove the Recurrence formulae :

    $$2xH_n(x) = 2nH_{n-1}(x) + H_{n+1}(x)$$

MAMT–03/MSCMT–03

June – Examination 2024

M.A./M.Sc. (Previous) Examination

MATHEMATICS

(Differential Equations, Calculus of Variation and Special Functions)

Paper : MAMT–03/MSCMT–03

*Time : **3** Hours*] [*Maximum Marks : **80***

Note :– The question paper is divided into three Sections A, B and C. Write answers as per the given instructions.

Section–A **8×2=16**

(Very Short Answer Type Questions)

Note :– Answer all questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries 2 marks.

1. (i) Define an Exact equation.

(ii) Write the general form of a second order P.D.E. with two independent variables x and y.

(iii) The λ-equation in Monge's method for solving P.D.E. :

$$r + 3s + t + (rt - s^2) = 1$$

(iv) The condition for which P.D.E. :

$$Rr + Ss + Tt + f(x, y, z, p, q) = 0$$

is parabolic will be ....

(v) What is a Harmonic equation ?

(vi) Define Singular Point for Legendre equation.

(vii) State Gauss theorem for hypergeometric functions.

(viii) What is the value of $P_n(1)$?

Section–B **4×8=32**

(Short Answer Type Questions)

Note :– Answer any *four* questions. Each answer should not exceed **200** words. Each question carries 8 marks.

2. Solve :

$$z^2dx + (z^2 - 2yz)dy + (2y^2 - yz - zx)dz = 0$$

3. Show that a surface passing through the circle $z = 0$, $x^2 + y^2 = 1$ and satisfying the differential equation $s = 8xy$ is $z = (x^2 + y^2)^2 - 1$.

4. Using the method of separation of variables solve the following equation :

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t}$$

Given that $u(x, 0) = 6e^{-3x}$.

5. A tightly stretched string which has fixed end points $x = 0$ and $x = l$ is initially in a position given by $y = k\sin^3\left(\frac{\pi x}{l}\right)$. It is released from rest from this position. Find the displacement $y(x, t)$.

6. Find the eigenvalues and eigenfunctions for the boundary value problem $y'' + \lambda y = 0$ under the boundary condition $y(a) = 0$ and $y(b) = 0$, $0 < a < b$; a and b are arbitrary real constants.

7. Solve :

$$x^2\frac{d^2y}{dx^2} + (x + x^2)\frac{dy}{dx} + (x - 9)y = 0$$

in series.

8. Write and prove Kummer's theorem.

9. Prove that :

$$(2n + 1)\, xP_n(x) = (n + 1)\, P_{n+1}(x) + nP_{n-1}(x)$$