10. Let $< f_n >$ be a sequence of bounded measurable functions defined on a set E of finite measure. If there exists a positive number M such that $| f_n(x) | \leq M$ for all $n \in N$ and for all $x \in E$ and if $< f_n >$ converges in measure to a bounded measurable function f on E, then prove that :

$$\lim_{n \to \infty} \int_E f_n(x) dx = \int_E f(x) dx$$

11. Prove that a subset of (R, U) is compact if and only if it bounded and closed.

12. Prove that every interval is measurable.

13. State and prove Riesz-Fischer theorem.

MAMT–02/MSCMT–02

June – Examination 2024

M.A./M.Sc. (Previous) Examination

MATHEMATICS

(Real Analysis and Topology)

Paper : MAMT-02/MSCMT-02

*Time : **3** Hours*] [*Maximum Marks : **80***

Note :– The question paper is divided into three Sections A, B and C. Write answers as per the given instructions.

Section–A **8×2=16**

(Very Short Answer Type Questions)

Note :– Answer all questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries 2 marks.

1. (i) Define Borel set.

   (ii) Define measurable function.

   (iii) State Weierstrass approximation theorem.

   (iv) State Minkowski's inequality.

   (v) Write the necessary and sufficient conditions for a bounded function f defined on the interval $[a, b]$, to be L-integrable.

   (vi) Define Hilbert space.

   (vii) Define exterior of a set.

   (viii) What do you mean by filter base ?

Section–B 4×8=32

(Short Answer Type Questions)

Note :– Answer any *four* questions. Each answer should not exceed **200** words. Each question carries 8 marks.

2. Let $\{E_n\}$ be a countable collection of sets of real numbers, then show that :

$$m^*\left(\bigcup_n E_n\right) \leq \sum_n m^*(E_n)$$

3. Show that every bounded measurable function f defined on a measurable set E is L-integrable.

4. If a function is summable on E, then show that it is finite almost everywhere on E.

5. Show that an orthonormal system $\{\phi_i\}$ is complete iff it is closed.

6. State and prove Holder's inequality.

7. Show that regularity is a topological property.

8. Show that the property of a space being a Hausdorff space is a hereditary property.

9. Prove that every open continuous image of a locally compact space is locally compact.

Section–C 2×16=32

(Long Answer Type Questions)

Note :– Answer any *two* questions. You have to delimit your each answer maximum up to **500** words. Each question carries 16 marks.