10. Solve the following system of equations by Newton-Raphson method by taking initial approximaiton $x_0 = 1$, $y_0 = 1$.

$$y - \sin(x + y) = 0$$
$$x - \cos(y - x) = 0$$

11. Write necessary condition for applying Cholesky's method. Using Cholesky (square root) method solve the system of equations :

$$4x - y = 1$$
$$-x + 4y - z = 0$$
$$-y + 4z = 0$$

12. Solve the following initial value problem :

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 0 \qquad t \in [0, 0.1]$$

$y(0) - 0$, $y'(0) - 1$

13. (a) Find the value of y at $t = 0.2$ by using seven terms Taylor's series, where $y(t)$ is the solution of the second order initial value problem :

$$\frac{d^2y}{dt^2} = 4 - t + y^2,\ y(0) = 1,\ y'(0) = -1$$

(b) Solve the boundary value problem by Numerov method with step size $h = \frac{1}{4}$.

$$\frac{d^2y}{dx^2} = x + y,\ y(0) = 0,\ y(1) = 0$$

MAMT-08/MSCMT–08

June – Examination 2023

M.A./M.Sc. (Final) Examination

MATHEMATICS

(Numerical Analysis)

Paper : III

Paper : MAMT-08/MSCMT-08

*Time : **3** Hours*] [*Maximum Marks : **80***

Note :– The question paper is divided into three Sections A, B and C. Write answers as per the given instructions. Use of non-programmable scientific calculator is allowed in this paper.

Section–A **8×2=16**

(Very Short Answer Type Questions)

Note :– Answer all questions. As per the nature of the question delimit your answer in maximum up to **30** words. Each question carries 2 marks.

1. (i) Write the difference between Secant method and Regula-Falsi method.

(ii) Write Newton-Raphson formula to find p^{th} root of a number.

(iii) Show that $x = 1$ is a multiple root of equation $x^3 - 3x^2 + 3x - 1 = 0$ with multiplicity three.

(iv) Define Spectrum and spectral radius of a matrix.

(v) Define Trace of a matrix.

(vi) Define tridiagonal matrix and give an example.

(vii) Write normal equations for fitting a parabola $y = a + bx + cx^2$.

(viii) State minimax property of Chebyshev's polynomial.

Section–B **4×8=32**

(Short Answer Type Questions)

Note :– Answer any *four* questions. Each answer should not exceed **200** words. Each question carries 8 marks.

2. Find complex root of the equation $x^3 - x^2 - x - 1 = 0$ by using Chebyshev method.

3. Using synthetic division and Chebyshev method find a root of the equation $x^3 + x^2 + 3x + 4 = 0$ Perform two iterations.

4. Use of two iterations of Jacobi method to compute eigenvalues of given matrix :

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

5. Using the Rutishauser method, find all the eigenvalues of the matrix :

$$A = \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$$

6. Explain least-squares principle for continuous functions.

7. Find a uniform polynomial approximation of degree or less to the function $f(x) = e^x$ on the interval $[-1, 1]$ using Lanczos economization with error tolerance 0.02.

8. Compute $y(0.5)$ by Milne's method, given that :

$\frac{dy}{dt} = 2e^t - y$ and the corresponding values of t and y are given as :

t	0	0.1	0.2	0.3
y	2	2.01	2.04	2.09

9. Solve the boundary value problem $\frac{d^2y}{dx^2} = y$, $y(0) = 0$, $y(1) = 1.2$ by employing shooting method, take $y'(0) = 0.85, 0.95$ as initial approximations.

Section–C **2×16=32**

(Long Answer Type Questions)

Note :– Answer any *two* questions. You have to delimit your each answer maximum up to **500** words. Each question carries 16 marks.