MAMT–10/MSCMT–10

June – Examination 2023

M.A./M.Sc. (Final) Examination

MATHEMATICS

(Mathematical Programming)

Paper : MAMT–10/MSCMT–10

*Time : **3** Hours*] [*Maximum Marks : **80***

Note :– The question paper is divided into three Sections A, B and C. Write answers as per the given instructions.

Section–A **8×2=16**

(Very Short Answer Type Questions)

Note :– Answer all questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries 2 marks.

1. (i) Define extreme point of a convex set.

(ii) Write down the standard form of a Bounded variable Linear Programming Problem (BVLPP).

(iii) Define a Integer Programming Problem.

(iv) Define a matrix notation of General Non-linear Programming Problem (GNLPP).

(v) Write the Hessian Matrix for the function :

$$f(x_1, x_2) = (x_1 - 2)^2 + (x_2 - 1)^2$$

(vi) Define a Convex Programming Problem.

(vii) Write $\frac{ds}{dz}$ for the function :

$$S = Z^2 + M\left(\frac{b}{z}\right)^{2/m}$$

(viii) Which method is used to solve discrete and continuous, deterministic problem ?

Section–B 4×8=32

(Short Answer Type Questions)

Note :– Answer any *four* questions. Each answer should not exceed **200** words. Each question carries 8 marks.

2. Show that a Hyperplane is a closed set.

3. Explain the bounded variable simplex method.

4. Solve the following LPP using Revised Simplex Method :

   Max. :

   $$Z = 2x_1 + x_2$$

   S.t. :

   $$3x_1 + 4x_2 + \leq 6$$

   $$6x_1 + x_2 \leq 3$$

   $$x_1, x_2 \geq 0$$

5. Write the quadratic form :

   $$Q_{(x)} = x_1^2 + 2x_2^2 - 7x_3^2 - 4x_1x_2 + 6x_1x_3 - 5x_2x_3$$

   in matrix form.

6. Use Lagrangian function to find optimal solution of the following non-linear programming problem :

   Min. :

   $$f(x) = -3x_1^2 - 4x_2^2 - 5x_3^2$$

   S.t. :

   $$x_1 + x_2 + x_3 = 10$$

   $$x_1, x_2, x_3 \geq 0$$

7. Determine whether or not the quadratic forms A^T AX are positive definite where :

   (a) $A = \begin{pmatrix} 1 & 3 \\ 0 & 4 \end{pmatrix}$

   (b) $A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

8. Derive the dual of the quadratic programming problem :

   Min. :

   $$f(X) = C^TX + \frac{1}{2}X^TGX$$

   S.t. :

   $$AX \geq b,\ X \geq 0$$

   Where A is an $m \times n$ real matrix and G is $n \times n$ real positive semi definite asymmetric matrix.

9. Find an optimal solution of the following convex separable problem :

Max. :

$$z = 3x_1 + 2x_2$$

S.t. :

$$4x_1^2 + x_2^2 \le 16$$

and $x_1, x_2 \ge 0$

Section–C 2×16=32

(Long Answer Type Questions)

Note :– Answer any *two* questions. You have to delimit your each answer maximum up to **500** words. Each question carries 16 marks.

10. (i) Prove that $f(x) = \frac{1}{x}$ is strictly convex for $x > 0$ and strictly concave for $x < 0$.

(ii) If $f(x)$ is continuous, $f(x) \ge 0$, $-\infty < x < \infty$, then the function :

$$\phi(x) = \int_x^{\infty} (y - x) f(y) dy$$

is a convex function provided the integral converges.

11. Use the Kuhn-Tucker conditions to solve the following non-linear programming problem :

Optimize :

$$f(x_1, x_2, x_3) = 2x_1 + 3x_2 - (x_1^2 + x_2^2 + x_3^2)$$

S.t. :

$$x_1 + x_2 \le 1$$

$$2x_1 + 3x_2 \le 6$$

$$x_1, x_2 \ge 0$$

12. Prove that :

(i) Every local maximum of the general convex programming problem is its global maximum.

(ii) The set of all optimum solutions (global Maximum) of the general convex programming problem is a convex set.

13. Solve the following LPP using dynamic programming :

Max. :

$$Z = 10x_1 + 30x_2$$

S.t. :

$$3x_1 + 6x_2 \leq 168$$

$$12x_2 \leq 240$$

$$x_1, x_2 \geq 0$$