MAMT–09/MSCMT–09

June – Examination 2023

M.A./M.Sc. (Final) Examination

MATHEMATICS

(Integral Transforms and Integral Equations)

Paper : MAMT–09/MSCMT–09

Time : **3** *Hours*] [*Maximum Marks* : **80**

Note :– The question paper is divided into three Sections A, B and C. Write answers as per the given instructions. Use of non-programmable scientific calculator is allowed in this paper.

Section–A **8×2=16**

(Very Short Answer Type Questions)

Note :– Answer all questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries 2 marks.

1. (i) Show that $f(t) = e^{t^3}$ is not of exponential order.

   (ii) Find Laplace transform of $\cosh^2 4t$.

   (iii) State Lerch's theorem for inverse Laplace transform.

   (iv) Define Fourier sine transform.

   (v) If $M\{f(x); p\} = F(p)$, then prove that $M\left\{\frac{1}{x} f\left(\frac{1}{x}\right); p\right\} = F(1-p)$.

   (vi) Write inversion formula for the Hankel transform.

   (vii) Define Fredholm integral equation of third kind.

   (viii) Define symmetric kernels.

Section–B 4×8=32

(Short Answer Type Questions)

***Note* :–** Answer any *four* questions. Each answer should not exceed **200** words. Each question carries 8 marks.

2. Prove that :

$$L\left[\int_t^\infty \frac{\cos u}{u} du; p\right] = \frac{\log\left(p^2+1\right)}{2p}$$

3. State and prove convolution theorem for Laplace transform.

4. Find the Fourier cosine transform of e^{-1^2}

5. Show that :

$$H_\nu\left\{\frac{d^2f}{dx^2}+\frac{1}{x}\frac{df}{dx}-\frac{\nu^2}{x^2}f; p\right\} = -p^2F_\nu(p)$$

where $F_\nu(p)$ is the Hankel transform of order ν of the function $f(x)$.

6. Solve :

$$g(x) = e^x + \lambda\int_0^1 2e^{x+1}g(t)\,dt$$

7. Solve the homogeneous Fredholm integral equation $\phi(x) = \lambda\int_0^1 e^{x+1}\phi(t)\,dt$

8. Prove that if a kernel is symmetric, then all its iterated kernels are also symmetric.

9. Solve the following integral equation :

$$g(x) = x + \lambda\int_0^1 \left(4xt - x^2\right).g(t)\,dt$$

Section–C **2×16=32**

(Long Answer Type Questions)

Note :– Answer any *two* questions. You have to delimit your each answer maximum up to **500** words. Each question carries 16 marks.

10. (a) Solve :

$ty'' + y' + 4ty = 0,\ y(0) = 3,\ y'(0) = 0$

(b) Solve :

$$\frac{\partial u}{\partial t} = 3\frac{\partial^2 u}{\partial x^2}$$

given that :

$$u_x(0,t) = 0,$$

$$u\left(\frac{\pi}{2}, t\right) = 0$$

and $u(x,0) = 30 \cos 5x$.

11. Define Mellin transform and state convolution theorem for Mellin transform. Prove that if m is a positive integer and $\alpha \neq 0$

$$M\left\{\left(x^{1-\alpha}\frac{d}{dx}\right)^m f(x); p\right\}$$

$$-(-1)^m \alpha^m \frac{\Gamma\left(\frac{p}{\alpha}\right)}{\Gamma\left(\frac{p}{\alpha}-m\right)} F(p - m\alpha)$$

where $M\{f(x);p\} = F(p)$.

12. Solve $\frac{\partial^4 U}{\partial x^4} + \frac{\partial^2 U}{\partial y^2} = 0, -\infty < x < \infty, y \geq 0$ satisfying the conditions :

(i) U and its partial derivatives tends to zero as $x \to \pm\infty$

(ii) $U = f(x), \frac{\partial U}{\partial y} = 0$ for $y = 0$.

13. Solve by iterative method :

$$g(x) = 1 + \lambda \int_0^{\pi} \sin(x+t)\, g(t)\, dt$$