10. State and prove Risez representation theorem Hilbert space.

11. If B and B′ be Banach spaces and T is a continuous linear transformation of B onto B′, then prove that the image of every open sphere centred at origin in B contains an open sphere centred at origin in B′.

12. Show that the set of all normal operators on a Hilbert space H is a closed subset of β(H) which contains the set of all self-adjoint operators and is closed under scalar multiplication.

13. State and prove inverse function theorem.

MAMT–06/MSCMT–06

June – Examination 2023

M.A./M.Sc. (Final) Examination

MATHEMATICS

(Analysis and Advanced Calculus)

Paper : MAMT–06/MSCMT–06

*Time : **3** Hours*] [*Maximum Marks : **80***

Note :– The question paper is divided into three Sections A, B and C. Write answers as per the given instructions.

Section–A **8×2=16**

(Very Short Answer Type Questions)

Note :– Answer all questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries 2 marks.

1. (i) Define weak convergence of a sequence.

(ii) What do you mean by conjugate space ?

(iii) Define Natural Embedding.

(iv) Write Taylor's formula with Lagrange's remainder.

(v) Define Self Adjoint Operator.

(vi) Define inner product space.

(vii) Describe the Quotient space.

(viii) Define the graph of a function.

Section–B **4×8=32**

(Short Answer Type Questions)

Note :– Answer any *four* questions. Each answer should not exceed **200** words. Each question carries 7 marks.

2. State and prove Minkowaski's inequality.

3. If x, y are any two vectors in a Hilbert space H, then prove that :

$$4(x, y) = \|x+y\|^2 - \|x-y\|^2 + i\|x+iy\|^2 - i\|x-iy\|^2$$

4. Prove that every Hilbert space is reflexive.

5. Let M be a linear subspace of Hilbert space H. Then prove that M is closed if and only if $M = M^{\perp\perp}$.

6. State and prove Schwarz inequality for an inner product space.

7. Let X be a Banach space over the field K of scalars and let $f : [a, b] \to X$ and $g : [a, b] \to R$ be continuous and differentiable functions such that $\|Df(t)\| \leq Dg(t)$ at each point $t \in [a, b]$. Then prove that :

$$\|f(b) - f(a)\| \leq g(b) - g(a)$$

8. State and prove global uniqueness theorem.

9. Show that every compact subset of a normed linear space is bounded but its converse need not be true.

Section–C **2×16=32**

(Long Answer Type Questions)

Note :– Answer any *two* questions. You have to delimit your each answer maximum up to **500** words. Each question carries 16 marks.