MAMT–05/MSCMT–05

June – Examination 2023

M.A./M.Sc. (Previous) Examination

MATHEMATICS

(Mechanics)

Paper : V

Paper : MAMT–05/MSCMT-05

*Time : **3** Hours*] [*Maximum Marks : **80***

Note :– The question paper is divided into three Sections A, B and C. Write answers as per the given instructions. Use of non-programmable scientific calculator is allowed in this paper.

Section–A **8×2=16**

(Very Short Answer Type Questions)

Note :– Answer all questions. As per the nature of the question delimit your answer in maximum up to **30** words. Each question carries 2 marks.

1. (i) Define product of inertia.

   (ii) Define centre of percussion and line of percussion.

   (iii) What do you mean by instantaneous axis of rotation ?

   (iv) Define invariable line.

   (v) State principle of conservation of linear momentum.

   (vi) What do you mean by steady motion of a top ?

   (vii) Define laminar flow.

   (viii) Define rotational and irrotational motion.

Section–B **4×8=32**

(Short Answer Type Questions)

Note :– Answer any *four* questions. Each answer should not exceed **200** words. Each question carries 8 marks.

2. Show that the centre of Inertia of a body moves as if all the mass of the body were collected at it and as if all the external forces were acting at it in directions parallel to those in which they act.

3. A rigid body is rotating about a fixed axis. Find the moment of the effective forces about the axis of rotation.

4. Deduce Euler's equations from Lagrange's equations.

5. A particle of unit mass is projected so that its total energy is h is a field of force of which the potential energy is $\phi(r)$ at a distance r from the origin. Deduce from the principle of energy and least action that the differential equation of the path is :

$$c^2\left[r^2+\left(\frac{dr}{d\theta}\right)^2\right]=r^4[h-\phi(r)]$$

6. Determine the stream line if the velocity of an incompressible fluid at the point (x, y, z) is given by :

$$\left(\frac{3xz}{r^5}, \frac{3yz}{r^5}, \frac{3z^3-r^2}{r^5}\right)$$

where $r^2 = x^2 + y^2 + z^2$.

7. Show that the ellipsoid :

$$\frac{x^2}{a^2k^2t^{2n}}+kt^n\left(\frac{y^2}{b^2}+\frac{z^2}{c^2}\right)=1$$

is a possible form of the boundary surface of a liquid.

8. State and prove Bernoulli's theorem.

9. A source S and a sink T of equal strength m are situated within the space bounded by a circle, whose centre is O. If S and T are at equal distances from O on opposite side of it and on the same diameter AOA′. Show that the velocity of the liquid at any point O is :

$$2m.\frac{\text{OS}^2+\text{OA}^2}{\text{OS}}\cdot\frac{\text{PA.PA}'}{\text{PS.PS}'\text{.PT.PT}'},$$

where S′ and T′ are the inverse points of S and T with respect to the circle.

Section–C 2×16=32

(Long Answer Type Questions)

Note :– Answer any *two* questions. You have to delimit your each answer maximum up to **500** words. Each question carries 16 marks.

10. A uniform rod of length $2a$ is placed with one end in contact with a horizontal table and is then at an inclination α to the horizon and is allowed to fall. When it becomes horizontal show that its angular velocity is $\sqrt{\frac{3g\sin\alpha}{2a}}$, whether the plane be perfectly smooth or perfectly rough. Show also that the end of the rod will not leave the plane in either case.

11. A lamina rotating with uniform angular velocity n about an axis through its centre of gravity perpendicular to its plane has an additional angular

velocity λn impressed upon it about the axis of least moments : (A < B < C), where $\lambda^2 = \frac{B+A}{B-A}$, Prove that time t its angular velocities are $\lambda n \sec h\, nt$, $\lambda n \tan nt$ and $n \sec h\, nt$. Also show that it will ultimately revolve about the axis of mean moment. Where M is the mass of the lower sphere and m is mass of the upper sphere.

12. Derive Lagrange's equations of motion in generalised coordinates for a holonomic dynamical system under finite forces.

13. A sphere of radius a is surrounded by infinite liquid of density ρ the pressure at infinity being π. The sphere is suddenly annihilated. Show that the pressure at a distance r from the centre immediately falls to $\pi\left(1-\frac{a}{r}\right)$. Show further that if the liquid is

brought to rest by impinging on a concentric sphere of radius $\frac{a}{2}$, the impulsive pressure sustained by the surface of this sphere is $(7\pi\rho^2/6)^{1/2}$.