Section–C **2×16=32**

(Long Answer Type Questions)

Note :– Answer any *two* questions. You have to delimit your each answer maximum up to **500** words. Each question carries 16 marks.

10. (a) Prove that the necessary and sufficient condition for the curve to be a straight line is that curvature is zero at all points of the curve.

    (b) Prove that the curve given by $x = a \sin u$, $y = 0$, $z = a \cos u$ lies on a sphere.

11. State and prove existence and uniqueness theorems for space curves.

12. (a) Prove that in general three lines of curvature pass through an umbilic.

    (b) A covariant tensor of first order has components xy, $2y - z^2$, xz in rectangular coordinates. Determine its covariant components in spherical polar coordinates.

13. Show that it is always possible to choose a coordinate system so that all the Christoffel symbols vanish at a particular point.

MAMT–04/MSCMT–04

June – Examination 2023

M.A./M.Sc. (Previous) Examination

MATHEMATICS

(Differential Geometry and Tensor)

Paper : IV

Paper : MAMT–04/MSCMT–04

*Time : **3** Hours*] [*Maximum Marks : **80***

Note :– The question paper is divided into three Sections A, B and C. Write answers as per the given instructions. Use of non-programmable scientific calculator is allowed in this paper.

Section–A **8×2=16**

(Very Short Answer Type Questions)

Note :– Answer all questions. As per the nature of the question delimit your answer in maximum up to **30** words. Each question carries 2 marks.

1. (i) Define Inflexional tangent.

(ii) Define principal normal and binormal.

(iii) Define conoid.

(iv) Define trajectory.

(v) Define surface of centres.

(vi) Write normal property of a geodesic.

(vii) Write Mainardi-Codazzi equation.

(viii) Define reciprocal tensor.

Section–B **4×8=32**

(Short Answer Type Questions)

Note :– Answer any *four* questions. Each answer should not exceed **200** words. Each question carries 8 marks.

2. Find the envelope of the family of planes :

$$F(x, y, z, \theta, \phi) \equiv \frac{x}{a}\cos\theta\sin\phi + \frac{y}{b}\sin\theta\sin\phi + \frac{z}{c}\cos\phi - 1 = 0$$

3. Show that on a right helicoid, the family of curves orthogonal to the curves $u \cos v$ = constant is the family $(u^2 + a^2) \sin^2 v$ = constant.

4. State and prove Meunier's theorem.

5. Show that conjugate direction at a point P on a surface are parallel to conjugate diameters of the indicatrix at P.

6. Derive canonical equations of a geodesic on the surface $\frac{1}{r} = \frac{1}{r}(u, v)$.

7. If surface of sphere is a two dimensional Riemannian space, compute the Christoffel symbols.

8. Prove that :

$$A^{ij}_{,j} = \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^j}\left(\sqrt{g} A^{ij}\right) + A^{jk}\begin{Bmatrix} i \\ jk \end{Bmatrix}$$

Show that last term vanishes if A^{ij} is skew symmetric.

9. Prove that the necessary and sufficient condition for a space VN to be flat is that the Riemann-Christoffel tensor be identically zero.